TRANSACTIONS
OF
THE OPTICAL SOCIETY.

Vol. XVIII. APRIL, 1917. No. 3.

Extraordinary Meeting, 12th April, 1917.

Held at Burlington House, London.

F. J. CHESHIRE, F.R.M.S., President, in the Chair.

The Minutes of the Extraordinary Meeting held on 10th February last were read, confirmed, and signed.

The notice convening the Meeting was read.

The PRESIDENT submitted the following alteration to Rule XXII., paragraph 3, proposed by the Council, which was unanimously adopted :—

"That paragraph 3 of Rule XXII. be deleted, and the following substituted for it :—'All papers communicated to the Society, with their illustrative drawings, accepted for reading and publication, also the copyright thereof, shall become the property of the Society, unless previous stipulation be made to the contrary. Papers read at the Meetings of the Society, or abstracts thereof, shall be published in such form as the Council shall determine.'"

The Extraordinary Meeting then terminated.

Ordinary Meeting, 12th April, 1917.

F. J. CHESHIRE, F.R.M.S., President, in the Chair.

The Minutes of the previous Ordinary Meeting were read, confirmed, and signed.

The following were balloted for, and duly elected to the Society :—

As Members.

WILLIAM REFORM ASHDOWN, Messrs. Heath and Co., New Eltham.
T. H. BALL, Mirror Manufacturer and Mercurial Silverer, London.
H. L. BENSKIN, Scientific Instrument Maker, Finchley.
JOSEPH CHARLES KIRKMAN, Lecturer in Physics at Swansea Technical College.
JOHN WILLIAM MASON, Messrs. Heath and Co., New Eltham.
G. MIELK, Lieutenant, Russian Army, and Optical Inspector, Russian Government Supply Committee, London.
EDWIN LOUIS PARR, Messrs. Heath and Co., New Eltham.
J. V. RAMSDEN, Lieutenant-Colonel, London.
J. B. REINER, Optician, London.
E. L. VINCENT, Optical Manufacturer, London.
ROBERT WIGGLESWORTH, Messrs. T. Cooke and Sons, York
FRANK WOOD, B.Sc., Glass Manufacturer, Barnsley.

It was announced that the undermentioned candidates had been approved by the Council for the ballot. The name of the proposer is appended :—

For the class of "Member."

ROBERT OSBORNE BURNS, A.R.T.S., Glasgow (F. J. Cheshire).
HARRY ARTHUR CARTER, London (P. F. Everitt).
JAMES GRAY DOCHERTY, B.Sc., Glasgow (F. J. Cheshire).
CHARLES FABRY, Paris (F. J. Cheshire).
ROBERT C. FISHER, London (G. W. Heath).
ANDREW ROBSON, B.Sc., Glasgow (F. J. Cheshire).

The following communications were then presented, those of Mr. Cochrane and Mr. Chalmers being taken as read. After the discussions on those of Mr. Martin and Dr. Carse, a vote of thanks to the Authors, proposed from the Chair, concluded the proceedings.

Light Filters for Eye Protection.

By L. C. MARTIN, A.R.C.Sc., D.I.C., B.Sc., Imperial College of Science and Technology.

Read and Discussed 12th April, 1917.

INDUSTRIAL developments, together with a great increase of travel, have of late years led to the necessity of seeking means to protect the eyes from the effects of intense or trying illumination. Glass workers and others find a necessity for such protection, as does the traveller in the glare of tropical sunlight. Investigations have been carried out to determine the causes of deleterious effects observed in particular cases, and many special glasses and light filters have been described for the mitigation of glare and strain. The present paper seeks to summarise the main results obtained, together with the considerations necessary in providing for particular circumstances, and also submits a series of spectrophotometric measurements on certain protective glasses and substitutes for glass.

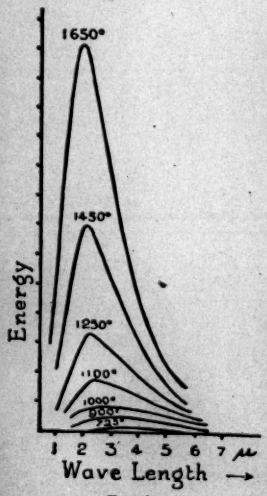


FIG. 1.

RADIATION.—A heated body when placed in space would become cold by reason of Radiation. According to the wave-theory, this action takes place by means of wave motions in the ether, and it is the wave-length or the frequency which determines the characteristics of the radiation. The shortest known waves are the X-rays, and with increasing wave-length come the ultra-violet, visible, infra-red, and lastly the electric waves used in wireless telegaphy.

Certain problems arise in screening the eyes from parts of the radiation emitted by heated bodies at various temperatures. The distribution of energy with wave-length, in the spectrum given by a full radiator, has been experimentally determined by Lummer and Pringsheim[1]. The wave-length corresponding to maximum energy, λ_m, is a function of the temperature, T, as are also the total energy emitted, E, and the energy at any particular wave-length, E_λ.

The formulæ are :—

$$\lambda_m T = \text{constant} = 2930 \text{ approximately},$$

$$E = c\,T^4$$

$$E = \frac{c_1 \lambda^{-5}}{(e^{c_2/\lambda T} - 1)} \text{ where } c, c_1, \text{ and } c_2 \text{ are constants.}$$

They have been theoretically derived, and agree fairly well with experimental data. They apply to full radiators only. In the case of substances giving bright line emission spectra, the maximum energy may, under certain conditions, lie in the ultra violet, as has been shown by Pflüger for the case of the condensed electric discharge between zinc electrodes. We have no evidence for believing, however, that the exceedingly high temperature, which such a condition would indicate if the emitting substance were a full radiator, has been attained.

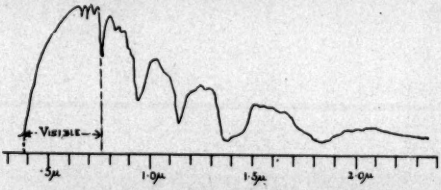
For melting glass at a temperature of 1400° C., say (absolute temperature at 1670° approximately), we shall have

$$\lambda_m = \frac{2930}{1670} = 1{\cdot}75\ \mu$$

For a Nernst glower

$$\text{Temp.} = 2200°\text{ C.}\ \lambda_m = 1{\cdot}2\ \mu$$

The point of maximum energy in the spectrum will thus lie far in the infra-red till very high temperatures are reached. The energy curve for sunlight has been experimentally determined by Langley[2]. One of his



LANGLEY'S NORMAL SOLAR SPECTRUM.

FIG. 2.

early results is shown. A great deal of work has been accomplished on this subject by the Smithsonian Institution. In the visible spectrum the Fraunhofer lines provide convenient points for reference. Wavelength is measured in terms of the unit μ which is equal to 10^{-4} cms. The Ångström unit sometimes used for the visible spectrum is 10^{-8} cms. For the sake of consistency we shall use the unit μ throughout the present paper.

	Wave Length.	
"Schumann" waves	0·10 μ	0·0001 mm.
Shortest solar waves reaching earth	0·29 ,,	0·00029 ,,
Ordinary glass transmits about	0·30 ,,	0·0003 ,,
Crookes ,, ,, ,,	0·36 ,,	0·00036 ,,
"Shorter" limit of visibility about	0·38 ,,	0·00038 ,,
Violet region ,,	0·40 ,,	0·0004 ,,
Blue ,, ,,	0·45 ,,	0·00045 ,,
Green ,, ,,	0·50 ,,	0·00050 ,,
Yellow ,, ,,	0·58 ,,	0·00058 ,,
Red ,, ,,	0·65 ,,	0·00065 ,,
"Longer" limit of visibility ,,	0·75 ,,	0·00075 ,,
Infra-red known waves ,,	107 ,,	0·107 ,,
Shortest "electric" waves ,,	6000 ,,	6·0 ,,

The visible spectrum represents a small interval of wave-length in comparison with the range of known radiations, but in the prismatic spectrum its importance is magnified, as the infra-red rays are "bunched" together when a beam of radiation is dispersed. In determining the distribution of energy in the spectrum, it is important to take this effect into account.

Especially in sunlight is the visible spectrum important. The point of maximum energy is at about ·46μ. The eyes in normal use should never receive the direct rays of the sun, but only scattered or reflected light. Blue light scattered from the sky has negligible infra-red energy, and the intensity becomes greater towards the blue end of the spectrum, but that reflected from haze, from the ground, from foliage, or even from water, may contain most wave-lengths originally present in the sun's rays.

1.—*Effects on the Eye.*

The physiological effects of radiation are not very fully understood, and the investigation of the subject is exceedingly difficult. The available evidence is not very full, but a summary may be useful.

It is a common experience that the eyes may be temporarily affected by exposure to an unshaded arc lamp. Under such conditions, fluorescence of the crystalline lens is caused, and, with long continued exposure, intense inflammation with final opacity of the media of the eye may result.

Experiments of Dr. E. K. Martin[3].—Absorption tests were made on the eyes of freshly killed young rabbits. It may be assumed that radiations which are not absorbed by the refracting media of the eye are incapable of harm, but if absorption takes place deleterious effects may follow.

This series of experiments deals firstly with the absorption of the ultra-violet, the method employed being to photograph the spectrum of

the iron arc with a quartz spectrograph, by light transmitted through a section of a part of the eye. Under these circumstances the spectrum generally is found to stop short at a certain wave-length. The results obtained were as follows :—

The Cornea absorbs waves shorter than ·295μ completely, and is fairly transparent to longer wave lengths.

The Crystalline lens begins to absorb at ·38μ, and absorbs completely at ·35μ.

The Vitreous (in a layer $\frac{3}{16}$″ thick) has an absorption band between ·25μ and ·28μ, but seems otherwise transparent.

The Crystalline lens, then, is the first to absorb, but it will not be reached by waves shorter than ·295μ. As the solar spectrum stops at ·29μ (about), it would appear that daylight is practically incapable of harming the Cornea.

In this connection, it may be well to remember that small absorption is difficult to detect by direct photographic methods.

A further series of experiments was concerned with the changes produced in the eyes of living animals by exposure to ultra-violet radiation from a mercury arc for varying periods.

In general, it was found that low intensities produced no permanent results. Corneal opacity resulted from radiation of great intensity, with conjunctivitis and other phenomena. Transmission of the radiation through benzol completely stops these effects, as the shorter waves are absorbed.

2.—*Physiological Effects of Strong Light.*

The foregoing experiments deal with the possible effect of ultra-violet radiation on the refracting media of the eye, but equally important is the consideration of the general effect of strong illumination. Distinction must first be drawn between permanent, or semi-permanent, chemical and structural changes, and harmful effects due to muscular strain, which may be produced by excessive contrast or brightness.

As is well known, the retina consists of a sheet of nervous tissue at the back of which are located the "rods and cones." These rest against the pigment cells, which play an important part in the machinery of adaptation. Fine extensions run up between the rods from these cells, which fill with the black pigment when the eye is exposed to light. This action is probably concerned in some way with the renewing of the photo sensitive material in the retina, and possibly, by absorbing light, the pigment effects the adaptation. The maximum sensitiveness of the eye is said to be a million times as great as the minimum value[4].

Direct sunlight focussed on the retina is capable of completely destroy-

ing the nervous structures, but such an extreme case is seldom met with. Excessive light may, however, become exceedingly exhausting, as is well known to travellers in tropical regions. It is established that the eyes of Southern races are more strongly pigmented than those of Northern descent, thus providing against the greater average intensity of illumination. If the light is so bright that after images persist for an appreciable time, then "glaring" effects and discomfort result. Snow blindness will be an extreme case.

Another effect of strong light is to cause a contraction of the iris, which to some extent relieves the excess of illumination on the retina; with a flickering light this effect is said to be a cause of strain[6].

A variety of so-called "glare" results from muscular efforts at accommodation or focussing when the eye seeks to observe moving objects, or to concentrate for some time on a *small* object. With low intensity of illumination these effects may be quite as apparent, but they must be remembered when the protection of the eyes is considered. In passing, the strain of searching the vacant field of a telescope (say) for some invisible object may be remembered. This strain may be noticed at night, and is certainly entirely muscular.

It seems to be generally admitted that contrast may have a considerable effect on vision. An American[4] authority states that vision is best with contrasts of about 1 : 20, and that no discomfort results with ratios up to 1 : 100. It may be remembered, however, that the sky viewed through a small window may give a contrast of 1 : 10,000. The impaired vision in such a case results evidently from the effect that the adaptation of the eye is not adjusted to the circumstances. To work with a small light while the rest of the room is darkened may lead to considerable strain; for the best vision a fairly even illumination of the retina is seemingly of importance.

Acuity of Vision.

In this connection it may be noticed that the eye is not achromatic. A very vivid illustration of this may often be observed in a theatre, where curious stereoscopic effects are given to objects at the same distance from the eye, but illuminated by lights of differing colour. In ordinary vision no chromatic faults are generally observed, as the apparent luminosity of the spectrum is very much greater in the yellow region than in the remainder, and the focussing is done for this point of maximum luminosity. The slight lateral spreading of the badly focussed images produced by other wave-lengths is not usually noticed, but it is of importance when the very finest vision is required. Here the provision of a filter, to reduce the intensity of the violet and red ends of the visible spectrum, considerably enhances visual acuity, as has been definitely

shown by Luckiesh in his recent book on colour. He set up a special test object, and took " acuity values " with time of exposure of the eyes, alternating with green and colourless glasses every three minutes. The result of the observations is shown in the diagram

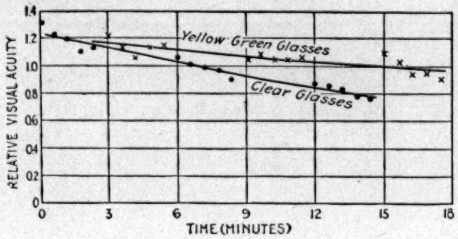


FIG. 3.

He states that the yellow-green glasses gave perceptible relief. Doubtless the improvement of vision is in some measure due to the reduction of light intensity, but also, probably, to the more nearly monochromatic nature of the light.

3.—*Effects of Infra-Red Rays.*

It has long been known that workers with blast furnaces, glass workers, and others, may develop cataracts in their eyes through continued exposure to heat radiations. This subject has been investigated by Sir W. Crookes. He has established that no " X " rays are emitted by molten glass, and that the radiation is chiefly confined to the longer wave-lengths.

Water is one of the most strongly absorbent substances for the infra-red. Thus it is practically certain that the refracting media of the eye would absorb infra-red radiation beyond about 1μ almost completely. The precise action of the radiations in causing opacity seems uncertain; it may be that the simple rise of temperature consequent on absorption produces the results observed. In any case, it seems clear that efficient protection from such dangers is highly important.

Summary and Criticism.

It has been conclusively shown that intense ultra-violet light is absorbed by the refracting media of the eye, and that it is then capable of producing physiological action. No absorption was detected for the visible region

of the spectrum, but it has not been proved to be entirely absent. Sir W. Crookes, and others, conclude that physiological actions follow the absorption of infra-red radiations, resulting in cataracts.

The effects of ordinary strong illumination are retinal and muscular Contraction of the iris, and flowing of the retinal pigment, follow exposure to light. No information seems to be available as regards any variation of these effects with the colour of the stimulating light. Dr. Kerr[6] states that except in cases of flicker this muscular contraction of the iris may probably be neglected as a cause of strain.

It seems that considerably more information is needed on these points. With regard to the contraction, it may be that the effect is due[15] partly to the stimulation of the nervous structures surrounding the iris by the absorption of light; such a mechanism would be extremely delicate, and here, again, information is needed as to the action of the various wave-lengths in producing the phenomenon. Certainly the adaptation thus produced is small compared with retinal adaptation, but it is not clear that the strain produced in these structures by excess of light is negligible.

Filters to absorb ultra-violet radiations are by no means a panacea for relieving eye-strain and glare. Again, it will not be contended that a bright blue sky is incapable of producing glare, although the amount of infra-red energy is negligible. The first importance of a filter will be to reduce the actual luminosity of the retinal image where glare is in evidence. Absorption of the ultra-violet and infra-red will be desirable precautions in ordinary circumstances, but of the highest importance under special industrial conditions.

Vision Through Telescopic Systems.

Very definite cases of eye strain arise through the use of telescopes, especially when their use is constant, for signalling or other purposes.

Suppose that the telescope is directed towards the brightest extended area which will commonly be met with, *i.e.*, the sky; also that the apparent brightness is I, then by a well-known theorem if

μ = index of refraction of the vitreous humour,
p' = radius of the pencil of light as it enters the eye after passing through the instrument,
v = distance of the unit plane of eye from the retina,

the brightness of the retinal image will be given by[7]

$$B = \frac{\mu^2 I \pi p'^2}{v^2}$$

If the eye looks towards the sky without a telescope, and p is the radius of the whole pupil, then the brightness B′ of the retinal image will be

$$B' = \frac{\mu^2 I \pi p^2}{v^2}$$

When the eye views the sky directly in the average way, let the illuminated area on the retina be A′; when it looks through the telescope, let the corresponding area be A; then the ratio of the total energy passing through the crystalline lens when using the telescope, to that when not using it, will be

$$\frac{A \cdot B}{A' \cdot B'} = \frac{A \cdot p'^2}{A' p^2}$$

Since in the first case, with the telescope, an area $\pi p'^2$ of the unit plane of the lens is concerned, and an area πp^2 in the second case, the ratio of the energy passing through unit cross sectional area of the lens in the two cases will be simply $\frac{A}{A'}$

With a telescope having a small field, a small patch on the retina will be illuminated, and the energy passing through the lens will be a small fraction of that received by an ordinary eye looking at the sky. Of course, if the telescope were turned towards any brighter source, such as the sun, the energy received by the lens would become correspondingly great.

In a prism binocular, the light may have to pass through a thickness of about 8 cm. of glass. The absorption of such a system was tested with an iron arc and quartz spectrograph, when the spectrum was found to stop short at a wave-length ·34μ. This points to an absorption of more than half of the supposedly harmful radiations in the ultra-violet.

Also, the theory shows that the brightness of illumination on the retina will generally be reduced when using the telescope.

It follows that the strain of using telescopic systems will seldom be due to any abnormally great intensity of light energy, although such cases arise when observing objects near the sun.

One or two possible causes of strain suggest themselves from the facts already reviewed.

(1) Contrast.—The illumination of the retina may be reduced to a small central patch surrounded by a darkened area. According to the general opinion, this is by no means the best condition for comfortable vision. Glare and strain may result from an illumination too intense for the *unadapted eye*.

(2) Bad Definition.—If this were present, the muscular system of the eye will be constantly seeking to focus by altering its

accommodation—an impossible task. For the best results no higher magnification should be used than is necessary, as the definition is generally not so good.

(3) VIBRATION OF THE IMAGE.—Any unsteadiness in the support of the instrument will lead to greater muscular eye strain through the motion of the telescope, and consequent shifting of the image.

The intensity of illumination of the field may be reduced by providing a suitable filter. This should stop ultra-violet well, and also a considerable proportion of the visible energy. If the telescope be used to view objects near the sun, some infra-red energy will be received, due to reflection from the haze. It would thus be a precaution to use a filter to absorb as much infra-red as possible. The use of a coloured filter to render the radiation more monochromatic will improve observation. Haze and excessive blue light may be avoided by suitably coloured filters, green, yellow and red being variously employed.

It will further be an advantage that the instrument should possess a large field of view, thus obtaining a broad retinal illumination. The provision of a stop to cut down the field to that part illuminated by full pencils will be, in this connection, a doubtful advantage.

Methods and Apparatus Employed for Testing Light Filters.

The present paper cannot deal fully with methods of spectrophotometry in general, but a short descripiton of the apparatus and methods used to obtain the results about to be given will perhaps be of interest.

(1) *Ultra-Violet Region.*

Visual methods are, of course, impossible, and recourse must be had to photographic, photo-electric, or other methods. For the present work the combined photographic and photometric apparatus, made by Messrs. Hilger, Ltd.,[9] was employed for a series of experiments, and a full description of the method will be found in their catalogues. The diagram shows the arrangement.

In effect, the object is to illuminate half the slit of a quartz spectrograph by radiations passing through a test specimen, and the other half by full radiation from the same source. In the path of each beam is a rotating sector (one of variable angle), by means of which the ratio of the exposures may be varied. In practice, the sector aperture for the unabsorbed beam is reduced.

When no specimen is present, and the aperture of both sectors is 90° a photograph taken of the spectrum should show the two spectra from each half of the slit side by side, and of exactly equal density. When

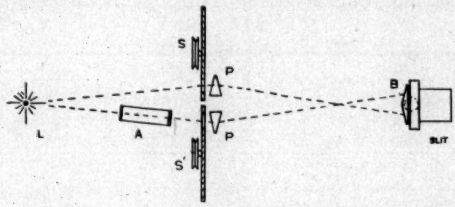


FIG. 4.

a specimen is in the path of one beam, and the exposure of the other is reduced in some known ratio, the adjacent spectra may give equal density on the plate at a certain measurable wave-length.

Quoting the catalogues, " Let the fractions of whole revolutions during which the variable and fixed aperture sectors allow light to pass be respectively t and t^1 and let the intensities of light of wave-length λ, reaching the slit by the upper and lower beams in the figure, be respectively I and I_1. Then, if at wave-length λ the density is exactly the same in both spectra, we have

$$\frac{I}{I_1} = f(t_1, t)$$

According to Schwarzschild, the relation for uninterrupted exposures is of the form,

$$\frac{I}{I_1} = \left(\frac{t^1}{t}\right)^n,$$

where n is a constant depending on the particular kind of photographic plate employed."

Messrs. Hilger supply with the apparatus a glass plate calibrated for absorption by another method based on the uniform transmission throughout the spectrum of a quartz plate, heavily silvered, and from which a certain area of silver has been removed by scratching. This glass plate is used to determine the constant n for the plates employed, the formula of Schwarzschild being assumed to hold.

In a recent number of the *Physical Review*, however, Professor Howe[10] describes experiments which he interprets as indicating that reliable results for absorption can be obtained direct from the ratio of the sector openings, but even if this be correct it is difficult to see where Messrs. Hilgers' method of getting the absorption of their standard glass plate

fails. Until we have more exact information on the subject it seems that results obtained must be used with caution. Considered relatively, however, they are extremely interesting as showing the behaviour of the variously coloured specimens, and the effects of their thickness.

Up to the present time it has been found necessary to employ a condensed discharge between uranium electrodes, or some such arrangement, for the source of light. A new departure was to use the "Pointolite" Tungsten arc-lamp of the Ediswan Co. This is quite satisfactory if over-run, as the nitrogen bands, which are somewhat troublesome and prominent, then become merged in continuous spectrum. This, of course, shortens the life of the lamp. For testing the absorption of glasses the spectrum given is sufficient, but it would scarcely be practical to use the lamp, even in a quartz bulb, for such work as this between $\cdot 2\mu$ and $\cdot 3\mu$.

For the purpose of this work, Professor Howe's deductions have been assumed to be correct, but the results can be modified if alteration is found to be necessary. Comparatively few results have been at present obtained with this apparatus, but it is hoped that their interest will justify their inclusion in the paper.

Limit of Transmission.

The coefficient of absorption of glasses becomes very high in the ultra-violet, and beyond a certain region for each specimen no transmission can, in general, be detected under standard circumstances. The thickness of the glass, intensity of the source, and the exposure, all affect the apparent limit of transmission, as does the construction of the spectrograph employed in the experiment. Consistent results under similar circumstances can, however, be obtained, and considerable information may be deduced regarding the transmission.

For the present work a small quartz spectrograph was employed; light from an iron arc was focussed on the slit by a quartz lens, and a photograph taken with each specimen in turn in front of the slit; five seconds exposure was given. From the resulting photographs the limits of transmission were found by means of a map of the iron spectrum. Comparing the results thus obtained with those given by the photometer, it will be seen that the "limit" corresponds to a transmission of something less than 1 per cent., or thereabouts, of the initial radiation.

(2) *Apparatus for the Visible Spectrum.*

I have been enabled to use this apparatus through the courtesy of Sir William Abney, who has already described it fully[11].

It consists of an apparatus to form a spectrum; in the plane of this

spectrum is placed a narrow slit, which thus emits fairly monochromatic radiations. A three-pole arc lamp (giving a very bright and steady light) is used as a source. The further arrangement is shown in the diagram.

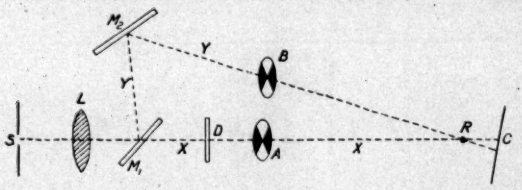


FIG. 5.

S is the slit moving in the spectrum, L the lens, which is so arranged as to throw the image of the first prism face in the spectrum apparatus on the screen C. A glass plate or bundle of plates, M_1, reflects a part of the beam to the second mirror M_2, whence it comes back to the screen C. Sectors can be inserted in one or both of the beams. A rod R is placed so as to give two shadows, touching one another at C. We thus have one-half of the screen illuminatd by one beam, and half by the other. The test specimen is placed in the path of the main beam, and the brightness of the shadows again equalised by adjustment of the sectors. A screen of metal coated with the white powder from burning magnesium is used throughout; the rest of the room must be kept in absolute darkness. The measurement may be made for light of any spectral colour by moving the slide on which the slit is mounted. A scale giving wavelengths is carried by the slide.

Extremely good and consistent results can be obtained by the use of this apparatus, which is naturally more accurate than spectrophotometers of the Nicol Prism type, where the results depend on the squares of the tangents of angles.

The fact that the apparent luminosity of a screen, illuminated by light interrupted by a rotatory sector, is proportional to the angular aperture of the sector has been well established by experiment. Consequently it is only necessary to equalise the two shadows before putting the specimen in place and after, when the transmission follows simply from the ratios of the openings.

(3) *Apparatus Used for the Infra-Red.*

I am indebted to Professor Callendar for permission to use this apparatus, which was specially designed by him some eight years ago

for infra-red work. It is essentially a mirror spectrometer with a rock-salt prism. The dispersed image of the slit is focussed in the plane of another slit immediately behind which is placed an exceedingly delicate

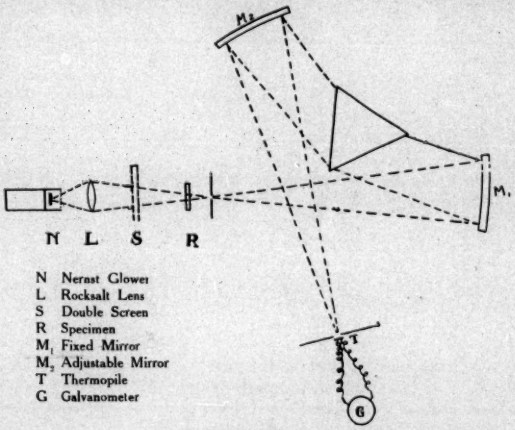


FIG. 6

thermopile. Means are provided for rotating the second mirror. This action moves the spectrum across the plane of the second slit, and in this manner exceedingly small portions of the visible or infra-red spectrum are allowed to fall on the face of the thermopile. The whole apparatus is enclosed in a heavily lagged box to prevent temperature disturbances.

In infra-red work it is, if possible, desirable to use a grating in order to obtain wave-lengths accurately, but in the present circumstances this was not possible. Accordingly, the following means were adopted for calibration :—

(1) The thermopile was displaced from behind the slit, and a reflecting prism was arranged so that the light passing through the slit could be viewed through a telescope.

(2) The rocksalt prism was rotated, and the mirror adjusted till the Lithium line (·6104) was at minimum deviation in the centre of the slit. Having given this arrangement, and knowing the angle of the prism, the angular distribution of the various wave-lengths can be calculated from the results given by Rubens [2].

(3) The lines (·5890) Sodium, (·6104) Lithium, (·6708) Lithium, were brought in turn to the centre of the field (with a very narrow slit), and the corresponding readings taken on the scale of the tangent action by means of which the mirror is rotated. The angular motion of the mirror for a given movement on this scale can easily be found.

(4) The thermopile is replaced and the slit widened. A strong Bunsen burner flame is now placed in front of the first slit. This gives an emission band at $4{\cdot}4\mu$[13], due to the heated carbon dioxide. The position of this band is well authenticated. It is now found on the tangent scale by rotating the mirror and observing the deflection of the galvanometer. Another point which may be found is the water absorption band very close to $3{\cdot}0\mu$. This band has been located by several experimenters[13] at slightly differing places. For purposes of the present work the absorption of a thin plate of selenite, which contains water of crystallisation and exhibits the band very well, was investigated. The position of this was found as accurately as possible and assumed to lie exactly at $3{\cdot}0\mu$.

(5) The angular motions for these wave-lengths, $3{\cdot}0\mu$ and $4{\cdot}4\mu$, were calculated from readings in (3) for visible lines, and from Rubens' results; the readings actually found being then used to apply a small correction, interpolating for the correction at intermediate wave-lengths.

In this manner, wave-lengths may be located with considerable accuracy on the assumption of the position of the water-band in the selenite plate. The results can be corrected if at any time this is shown to be wrong.

Having thus a means for measuring the intensity of radiation in different parts of the spectrum, the absorption of substances can be obtained provided that a steady source of radiation is available. It was desired to investigate the region from $\cdot7\mu$ to 5μ, and for this purpose a Nernst glower gives a fairly satisfactory emission, but is liable to be unsteady in ordinary circumstances. After many trials, the arrangement finally adopted was to enclose the lamp in a chimney of mica, with a thin quartz window; the glower is thus kept quite free from draughts. It is important also to ensure that the current taken shall be constant. For these experiments a spare set of accumulators which give a very constant E.M.F. was used. These precautions are extremely important. The image of the filament was focussed on the slit by means of a rocksalt lens, the whole

arrangement being made very steady. The focussing was varied till the deflection at the *longer* wave-length was as great as possible, which position, of course, was entirely different from the focus for the visible rays. This helps to keep the galvanometer deflections a little more uniform throughout the spectrum. A further trouble was met in parasitic E.M.F.'s in the thermopile circuit, which necessitated taking the zero for every two readings. For a long time the zero was taken by lowering a screen immediately in front of the thermopile, but this was found to vary slightly in temperature from the pile itself, and consequently a false zero was indicated. This, unfortunately, entailed the putting on one side of a great deal of work which had already been done. Finally, however, by the careful observation of these several precautions, consistent results could be obtained. For the purpose of this work it is not sufficient merely to locate the position of the absorption bands in the spectrum, which has been the object of most work done in the infra-red ; the exact numerical value of the proportion of energy for various wave-lengths which is transmitted must be found.

The secondary corrections for error due to non-parallelism of the beam passing through the test specimen, also for the small finite range of the spectrum allowed to fall on the thermopile have not been judged of sufficient importance for special notice in these experiments on absorption. Experimental details diminished errors due to such effects as far as possible.

General Remarks on the Methods and Results.

For all purposes the nature and position of the absorbing specimen is of importance. It must have good plane surfaces well polished. Bad surfaces can easily introduce a 20 per cent. error.

In the first two methods, variation of the source is allowed for by the simultaneous action of two beams, but extreme steadiness is of great importance in infra-red work.

The limits of accuracy of the measurements in the ultra-violet cannot at present be indicated, as sufficient experience of the method has not been gained. In the visible spectrum, results should be accurate to 1 in 100 at least. In the infra-red, where absolute consistency is even more difficult, results being obtained with different calibrations of the apparatus, and fresh adjustments of the radiator, lens and specimen, a general agreement to 2 or 3 in 100 is shown. The extreme results in the infra-red (wave-lengths 0·73 and 4·6μ) are liable to greater uncertainty, as here the actual galvanometer deflections do not exceed 3 or 4 mm. for the unabsorbed radiation.

Absorption of Radiation.

It can be shown from theoretical considerations that an electromagnetic wave, propagated in a material substance, will in general part with some of its energy to the electrons present, and this conclusion is, of course, confirmed from experience. The energy may re-appear as a re-emission of light in resonance radiation, fluorescence, or phosphorescence; it may become apparent in chemical action, in a rise of temperature, or in other ways.

The theoretical law states that if I_0 is the initial value of the energy in a light wave, the energy after passing through a thickness t of the medium, will be

$$I_t = I_0 e^{-at},$$

a is known as the coefficient of absorption.

This law holds for all homogenous transparent substances such as the glasses to be dealt with later; no attempt must be made to apply it to any such cases as stained light filters of celluloid. Stated in a simple form, the law shows that if a first thickness t of a material transmits a fraction a of the incident radiation, then the second thickness t will transmit the same fraction of the radiation incident upon *it*; a^2 will be the fraction transmitted by the two.

Theoretical considerations, again verified by experiment, give the intensity of the *reflected* light for normal incidence on a plane surface as

$$\left(\frac{n-1}{n+1}\right)^2$$

of the incident light where n is the refractive index. This corresponds to a transmission factor

$$= \left\{1 - \left(\frac{n-1}{n+1}\right)^2\right\} = (1-\beta), \text{ say.}$$

Then if I_0 is the incident radiation

Amount transmitted by 1st face	$= I_0(1-\beta)$	
,, ,, thickness of material	$= I_0(1-\beta)e^{-at}$	
,, ,, 2nd face	$= I_0(1-\beta)^2 e^{-at}$	

There will also be multiple reflections giving rise to further amounts emerging in the first direction with the transmitted beam,

$$I_0(1-\beta)^2\beta^2 e^{-3at},$$
$$I_0(1-\beta)^2\beta^4 e^{-5at},$$
$$+ \text{ etc.}$$

The total amount transmitted

$$= I_o(1-\beta)^2 e^{-at} + I_o(1-\beta)^2\beta^2 e^{-3at} + I_o(1-\beta)^2\beta^4 e^{-5at}, + \text{etc.}$$
$$= I_o(1-\beta)^2 e^{-at}\{1 + \beta^2 e^{-2at} + \beta^4 e^{-4at}, + \text{etc.}\}$$
$$= \frac{I_o(1-\beta)^2 e^{-at}}{(1-\beta^2 e^{-2at})}$$

The values of the refractive index in the visible and infra-red for various glasses have been determined by Rubens[8]. He shows that for most glasses the refractive index falls slowly with increasing wave-length. Hence β, which is about $\frac{1}{25}$ for ordinary glass, will steadily diminish. The term $\beta^2 e^{-2at}$ may thus be neglected for most purposes, and the formula becomes

$$I_1 = I_o(1-\beta)^2 e^{-at}$$

The formula may safely be used for glass till the region of quasi-metallic reflection is reached, when the substance will appear to be quite opaque.

Having measured the fraction $\frac{I_1}{I_o}$ for a plate of thickness t, we can make an allowance for the reflection by assuming a value for β. Thus, if the refractive index be 1·5,

$$\beta = \left(\frac{\cdot 5}{2\cdot 5}\right)^2 = \frac{1}{25}$$

We shall have

$$\log \frac{I_1}{I_o} = \log (1-\beta)^2 - at$$

We can calculate a and thence find the transmission for any other thickness of the material. If $\beta = \frac{1}{25}$, the practical formula will be

$$\log_{10} \frac{I_1}{I_o} = \log_{10} 0{\cdot}92 + Kt$$

By *measuring* the transmission of a plate of another thickness we can dispense with an assumed value for β; we shall get the value of

$$I' \text{ (say)} = I_o(1-\beta)^2 e^{-at'}$$

then

$$\frac{I_1}{I'} = e^{a(t'-t)}, \text{ whence } a \text{ may be found.}$$

For ordinary purposes we may work to the logarithm base 10 for convenience, the coefficient[16] in this case being known as the "extinction-coefficient." This short account of the theory will enable the results to be given to be transformed for other thicknesses.

In many scientific papers it is usual to calculate the value of the extinction-coefficient, and to give tables of this quantity. For the purposes of this work, however, it has been judged more useful to give the actual percentage transmission of definite specimens, with their thicknesses. Results can be transformed if necessary, but the curves will give a better idea of the actual performances of light filters than " extinction-coefficient " diagrams.

In order to summarise the results, the areas under the percentage curves for a given range of wave-length, expressed as fractions of the whole rectangle for that range, will give a measure of the general performance of a filter. This has been done for most of the specimens in the visible and infra-red regions. See table on page 91.

Composition of the Filters Tested.

(1) NEUTRAL GLASSES.—These are best produced by the use of nickel, manganese, iron, cobalt, and other oxides in amounts varying with special circumstances. The exact origin of the filters tested is unfortunately not known, but very similar properties are shown by glasses from three different sources.

(2) " CROOKES " SPECTACLE GLASS.—This is made by Messrs. Chance, and its exact composition has not been published. Its approximate composition may, however, be inferred from the paper in the " Phil. Trans. " already referred to.

Glass 202 contains ferric iron and cobalt.

Glass 246 contains ferrous iron.

(3) FIEUZAL GLASSES.—These have green tints probably due to chromium. They show the increased transmission due to Cr. at the red end of the spectrum noticed by Zsigmondy[14].

(4) HALLAUER GLASSES.—These are of German origin, supposed to be a combination of a green with a smoke tint.

(6) SIGNAL GREEN GLASSES.—These are soda lime glasses with varying amounts of cupric oxide.

(7) YELLOW AND RED GLASS SERIES.—These colours may be produced by the addition of varying amounts of nickel with other substances, also by the use of sulphur or selenium.

(8) THE CELLULOID SPECIMENS were strongly coloured by dye compositions.

TABLE OF SPECIMENS TESTED, SHOWING LIMITS OF TRANSMISSION IN ULTRA-VIOLET, AND RELATIVE ENERGY TRANSMISSIONS IN THE VISIBLE AND INFRA-RED.

SPECIMENS.	Thickness in mms.	Limit of Transmission in Ultra-Violet.	Relative Energy‡ Transmitted.	
			Visible. .427μ—.638μ.	Infra-Red. .91μ-4.6μ
Neutral glasses (graded) A	2.72	.32μ	63	53
Neutral glasses (graded) B	2.34	.30	28	50
Neutral glasses (graded) C	1.9	.365	3.5	29
Neutral (spare) glass D	2.52	.352	16†	48†
Do. " Crookes " B tint glass	3.8	.360	43† approx.	60†
" Crookes " Spectacle glass	6.92	.360	85† approx.	65†
Fieuzal glass (graded) 1	1.15	.340	77*	77*
2	1.11	.355	66	72
3	1.18	.365	61	66
4	1.23	.390	54	76
5	1.15	in the visible	48	74
6	1.26	,,	41	73
7	1.05	,,	32	74
8	1.18	,,	23	68
9	1.18	,,	15	63
10	1.26	,,	10	57
Hallauer glass (graded) 1	1.23	.340	88	74
2	1.10	.340	78	74
3	1.14	.385	58	74
4	1.24	in the visible	31	72
5	1.06	,,	23	67
Light green (spare) glass	2.47	.350	54†	62†
" Crookes " 202 glass	2.89	.352	27	22
,, 246 ,,	1.5	.360	20	7
Signal green 1 ,,	2.03	.347	26	40
,, ,, 2 ,,	1.71	.352	19†	30†
Yellow glass (spare) 1	2.32	.322	51†	54†
Yellow to Red grades 2	2.42	.325		
Yellow to Red grades 3	2.98	in the visible		
Yellow to Red grades 4	2.51	,,		
Yellow to Red grades 5	2.01	,,		
Celluloid — American non-flam., green	.26	.340	17	47
,, Collodion coated, n.f., blue	.43	.320	2	38
,, French, n.f., blue	.35	.295	6	39
,, ,, ,, blue	.32	.295	7	40
Gelatine, collodion covered, blue	.26	.360	2	26

REFERENCES.—† Value calculated for a thickness = 2mm.

* Ranges of wave length for Fieuzal and Hallauer glasses were .427μ—.692μ and .73μ to 4.6μ for visible and infra-red

‡ The energy transmitted by a filter depends on the source of radiation.

ULTRA-VIOLET TRANSMISSION. HILGER SPECTROMETER RESULTS. TABLE SHOWING WAVE-LENGTHS FOR GIVEN PERCENTAGE TRANSMISSIONS.

WAVE-LENGTH IN TERMS OF μ.

SPECIMEN.	63%	40%	32%	25%	20%	16%	10%	5%	3%	1%
Neutral glass A		.375	.358	.348	.339		.336		.328	
B			.355		.332	.323	.318	.315	.312	
C							.395	.385	.376	
"Crookes" Smoke Tint					.385	.378	.373	.366	.364	.362
" Spectacle glass	.400	.386		.380		.375	.373	.368	.366	.364
Fieuzal glass 1	.403	.388		.368		.364	.359		.350	
2		.406		.395		.389	.384		.369	
3				.407		.400	.393		.387	
4							.409		.401	.393
5									.410	.401
Hallauer glass 1		.403		.365		.352	.348		.345	.340
2		.357		.353		.348	.347		.340	.336
3									.410	.396

VISIBLE SPECTRUM. PERCENTAGE TRANSMISSION AND WAVE-LENGTH.

	·427μ	·444μ	·457μ	·492μ	·555μ	·589μ	·638μ	·692μ
Neutral A	59			59	63	66	66	
B	28			28	28	30	28	
C	3.5			4.0	4.6	3.0	2.3	
D	15	14		19	14	14	12	27
Crookes B		20		24	31	16	27	
" A		58		68	72	28	78	
Fieuzal 1	60	67	67	82	80	79	75	85
2	41		50	65	76	72	69	70
3	14		40	61	72	70	66	65
4	12		30	54	64	66	59	74
5	7		19	42	58	60	52	66
6	2.5		11	37	51	54	47	62
7	0.7		4.7	24	41	43	38	53
8	0.4		2.5	17	32	34	27	45
9	0.15		0.68	8.1	21	23	17	31
10	—		0.07	3.4	14.7	16	11	25
Hallauer 1	74		85	91	88	87	87	85
2	65		71	81	82	80	77	78
3	19		34	58	65	65	63	76
4	3.0		8.3	25	40	39	32	53
5	0.4		1.7	16	32	33	26	44
Light Green	30		35	50	69	65	49	50
Crookes 202	17		29	31	32	23	21	
" 246		16		22	20	16	12	
Signal Green 1	36		49	45	21	7.3	1.6	<1.0
2	25		38	46	16	5.0	<1.0	<1.0

VISIBLE SPECTRUM. PERCENTAGE TRANSMISSION AND WAVE-LENGTH—*contd.*

		·427μ	·444μ	·457μ	·492μ	·555μ	·589μ	·638μ	·692μ
Yellow to Red	1.. ..	11		19	38	66	75	76	70
	2.. ..	45			65	73	83	84	
	3.. ..		0.4		4.1	30	44	54	
	4.. ..		0.0		0.4	12.5	24	35	
	5.. ..		0.0		0.0	3.3	0.9	17	
Celluloid	1.. ..	12		7.8	14	22	25	9.4	
	2.. ..	1.7		2.9	3.4	1.3	0.30	0.0	
	3.. ..	6.1		4.9	7.4	7.0	5.2	2.9	
	4.. ..	3.7		8.4	14	7.3	2.3	0.0	
Gelatine	5.. ..	3.0		3.3	3.5	0.7	0.2	0.75	

INFRA-RED SPECTRUM. PERCENTAGE TRANSMISSION AND WAVE-LENGTH.

Wave-length (μ)	.90	1.43	2.05	2.47	3.0	3.56	4.18	4.54		
Neutral A	79	76	79	80	33	21	20	17		
B	66	70	66	72	34	26	30	22		
C	30	29	36	46	21	18	22	15		
D	52	58	60	67	44	37	35	9.7		
" Crookes B "	64	68	65	66	18	9	17	12		
" Crookes A "	60	72	76	70	7.7	2.7	12	12		
Light Green	79	86	86	85	49	39	35	24		
Signal Green 2	1.2	10.5	54	67	43	32	29	12		

Wave-length (μ)	.73	.91	1.28	1.67	2.65	3.15	3.55	3.95	4.3	4.6
Fieuzal 1	88	82	82		88		70	71	63	42
2	73	67	69	79	81		67	68	62	45
3	60	58	60	71	76		64	65	60	44
4	82	82	85	84	85		68	68	63	46
5	73	78	82	81	81		68	69	67	45
6	76	77	79	78	78		66	66	63	43
7	73	76	79	77	78		69	69	64	47
8	62	70	75	72	74		64	65	58	34
9	55	64	68	65	66		58	59	58	47
10	53	58	63	59	62		54	55	50	29
Hallauer 1	75	82	82	84	83	76	69	64	57	49
2	85	83	83		83	73	68	63	54	47
3	78	89	89		82	72	66	63	55	44
4	71	79	83		78	69	65	63	58	27
5	62	61	64		72	68	63	62	57	47
Crookes 202	34	19	15	33	38	10		8	5	
Celluloid 1	77	82	87	79	50	15		23	26	25
2	69	80	79	76	34	7		11	14	7
3	29	59	77	72	43	7		14	15	27
4	26	61	72	70	45	9		14	22	28
Gelatine 5	12	6	50	58	31	3		4	5	8

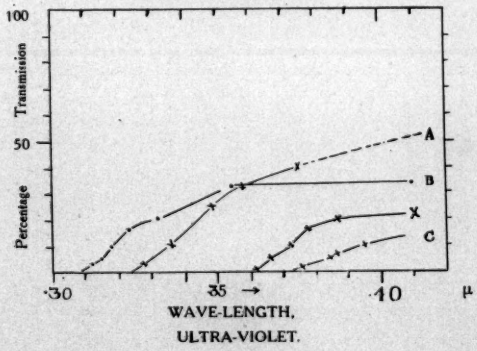
INFRA-RED SPECTRUM. PERCENTAGE TRANSMISSION AND WAVE-LENGTH—*contd.*

Wave-length (μ)	.92	1.32	1.5	2.04	2.53	3.04	3.46	3.87	4.24	4.60
Signal Green 1	3.9	10	22	61	73	49	38	39	30	12
Wave-length (μ)	.77	1.2	1.58	2.1	2.55	3.28	4.1			
Crookes 246	5.0	0.8	1.6	4.3	6.9	8.4	14.			
Wave-length (μ)	.90	1.43	2.05	2.47	3.0	3.56	4.18	4.5		
Yellow glass A	66	67	75	75	49	34	36	15		
Wave-length (μ)	.80	1.05	1.43	2.05	2.47	3.0	3.44	4.13	4.54	
Red tint E	52	51	60	76	74	55	43	27	19	

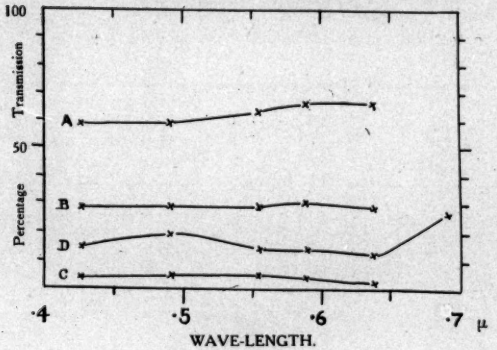
Analysis of Results. Diagrams Showing Percentage Transmission and Wave-Length.

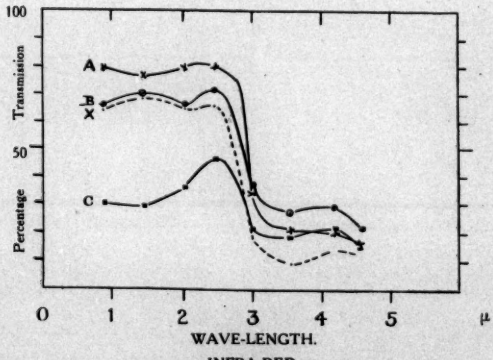
(1) NEUTRAL GLASSES.—Note the effect of thickness on ultra-violet transmission. B transmits more than A from .30 to .35. When only 3 or 4 per cent. of light is transmitted, transmission stops at .36. In the visible there is little to notice. Few glasses show no variation in per-

NEUTRAL GLASSES.



NEUTRAL GLASSES.





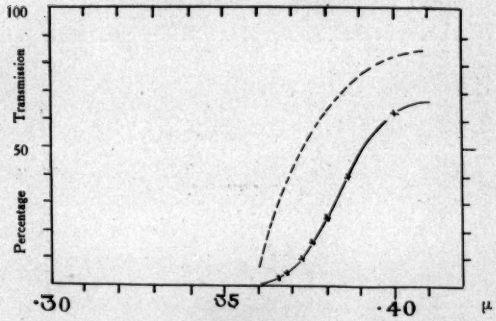
A B C, Graded Tints; D, Spare Specimen; X, Crookes' 'B' Tint.

centage transmission. "Crookes" B tint is not shown for the visible region, as in common with the "Crookes" ordinary (white) glass it has an absorption band, due to didymium, the exact form of which has not been investigated.

In the infra-red the effect of the colouring substance falls off towards the longer wave-lengths. Note the transmission of C, which rises to equality with A at about 4μ. This effect is very general throughout the glasses tested. X absorbs more at long wave-lengths owing to the greater thickness of the specimen.

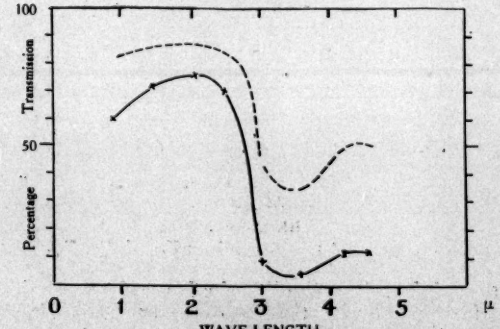
CROOKES' GLASS

Dotted Curve .. thickness, 2 mm.
Line „ .. „ 6·9 mm.



WAVE-LENGTH.

ULTRA-VIOLET.



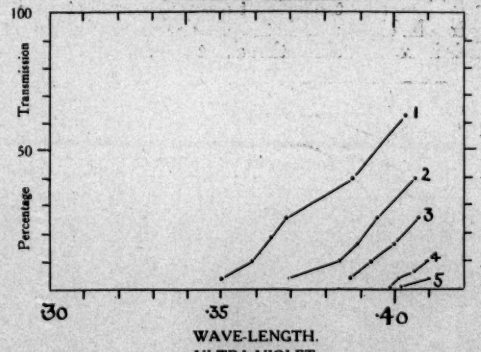
WAVE-LENGTH.

INFRA-RED.

(2) "CROOKES" GLASS (A TINT).—At a thickness of 6.92 mm. this glass transmits well in the infra-red, but absorbs heavily in the ultra-violet. There is no measurable transmission after .36μ. The results have been reduced by calculation to a value for 2 mm., and the resulting curve is shown for the ultra-violet and infra-red regions. The following table shows the effect of change of thickness.

	Transmission per cent.			Transmission per cent.	
Wave-length.	T = 6.92 mm.	T = 2 mm.	Wave length	T = 6.92 mm.	T = 2 mm.
.36 μ	Limit.		.90 μ	60 %	81%
.364 ,,	1.0%	25%	1.43 ,,	72 ,,	86 ,,
.38 ,,	25 ,,	63 ,,	2.05 ,,	76 ,,	87 ,,
.40 ,,	63 ,,	83 ,,	2.47 ,,	70 ,,	85 ,,
.444 ,,	58 ,,	80 ,,	3.0 ,,	7.7 ,,	45 ,,
.492 ,,	68 ,,	84 ,,	3.56 ,,	2.7 ,,	33 ,,
.555 ,,	72 ,,	86 ,,	4.18 ,,	12 ,,	51 ,,
.589 ,,	28 ,,	65 ,,	4.54 ,,	12 ,,	51 ,,
.638 ,,	78 ,,	88 ,,			

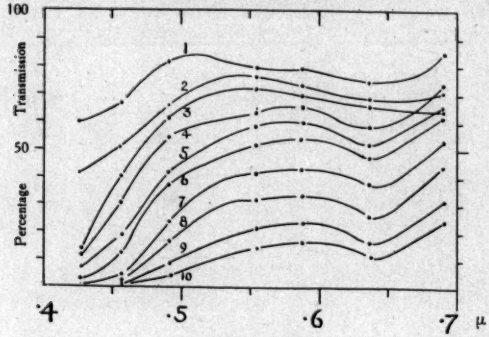
FIEUZAL GLASSES.—The absorption of the ultra-violet rapidly increases with depth of colour. When the visible energy is halved, the transmission



limit is in the visible spectrum. The colour is a pleasant green, becoming more yellow with increasing density. Note the shift of maximum transmission in the visible.

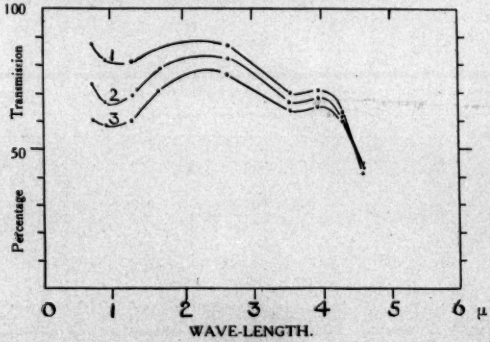
It would be interesting to know why Nos. 1, 2, and 3 are so markedly different from the rest in the region 1μ. From the position of the maxi-

FIEUZAL GLASSES.



WAVE-LENGTH
VISIBLE.

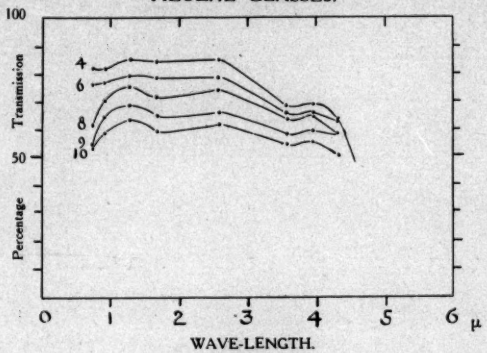
mum in the visible, and the evident absorption band at 1μ, it looks as if copper or iron had been used in these. The characteristic of chromium glasses is their increase of absorption about .65μ.



WAVE-LENGTH.
INFRA-RED.

Note the gradual approach of the curves to the same point at long wave-lengths in the infra-red. Little absorption is effected in this region.

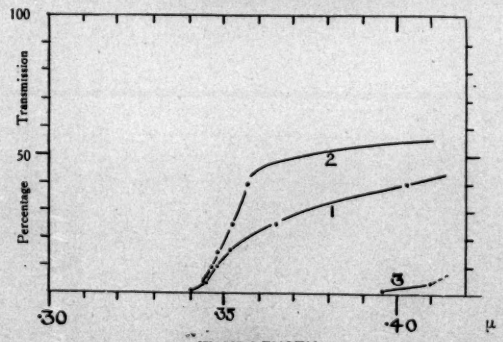
FIEUZAL GLASSES.



WAVE-LENGTH.
INFRA-RED.

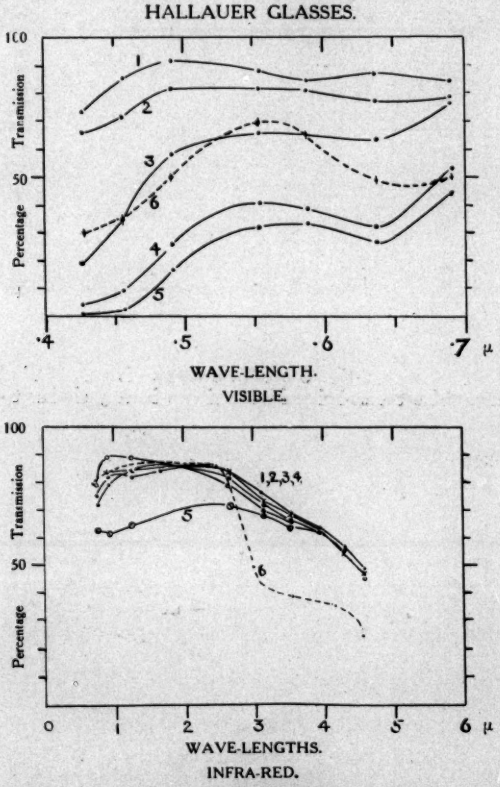
HALLAUER GLASSES.—The effect of thickness is very marked in the ultra-violet. The transmission in the visible is very similar to that of

HALLAUER GLASSES.



WAVE-LENGTH.
ULTRA-VIOLET.

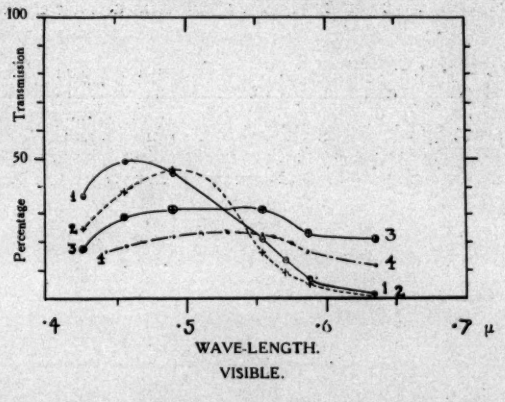
the Fieuzal series. There is no marked infra-red absorption beyond that due to the glass itself. The " light green " (No. 6) is more than twice as thick as the Hallauer tints, and shows greater absorption at long wave-lengths.

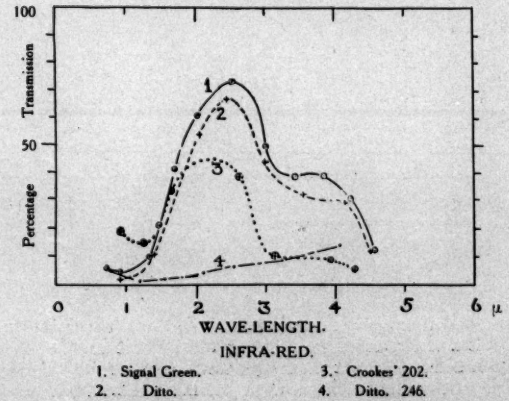


1, 2, 3, 4, & 5—Hallauer graded tints.
6—Spare glass (light green).

"CROOKES" 202.—This contains ferric iron and cobalt. The maximum absorption seems near 1.3μ in the infra-red. The limit in the ultra-violet is .352μ, but this is with a fair thickness. The infra-red

CROOKES' GLASSES.





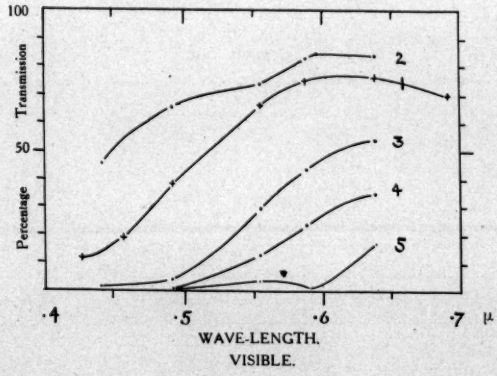
1. Signal Green.
2. Ditto.
3. Crookes' 202.
4. Ditto. 246.

absorption is good compared with most other glasses, and keeps the transmission down between 1.5 and 3.0μ.

"CROOKES" 246.—The absorption band at about 1.3μ is very strong, owing, presumably, to considerable amounts of ferrous iron. This glass stops practically all the infra-red, while transmitting about 20 per cent. of visible energy. It is not very heavy in colour, and absorbs the ultra-violet very well.

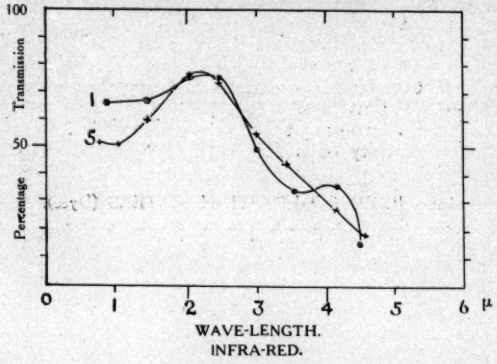
SIGNAL GREEN.—Results for two specimens are shown. The colour of one was deeper and more blue than the other. The chief colouring agent used for "signal green" is cupric oxide, and a characteristic absorption at about 1.0μ is shown. This is not so intense as the ferrous iron absorption, but would make it a very useful glass for wear in sunlight, as the band is in a region of great heat energy in the solar spectrum.

TINTS YELLOW TO RED.



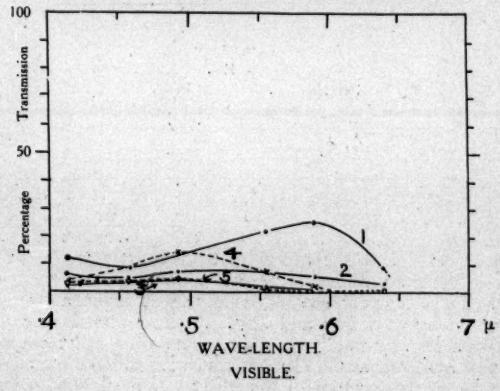
YELLOW-RED SERIES.—The lightest of this series transmits well down into the ultra-violet although reducing the intensity. The others have a maximum transmission at the longer wave-lengths in the visible. None of the series show any particular absorption in the infra-red.

TINTS YELLOW TO RED.



CELLULOID FILTERS (DYED).—The behaviour of these is very similar to that of glass. Strong absorption at about 3μ is shown by many organic and other compounds, notably by water, and alcohol. The transmission

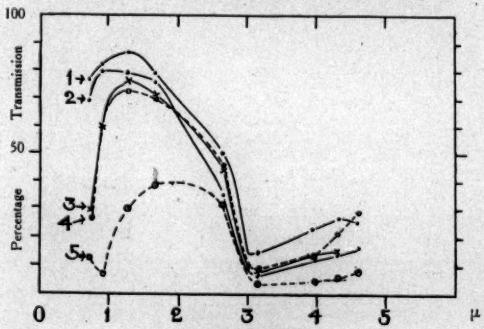
CELLULOID COMPOSITION FILTERS (Dyed).



in the visible and ultra-violet is small, although the limit is at a somewhat short wave-length. The collodion coated gelatine alone shows considerable absorption in the infra-red.

There are a few special devices which may provide useful filters. Metallic films on glass are the chief of these. A gold film has an almost uniform transmission, except in the green, where the transmission rises. It is thus possible to get a film reflecting nearly all the infra-red and visible, except in this region. The ultra-violet is similarly reduced. The difficulty of making such films which, for gold, must

CELLULOID COMPOSITION FILTERS (Dyed).



1. American Non flam. Celluloid (Green).
2. Blue ,, ,, (Collodion covered).
3. French ,, ,, (Blue).
4. French ,, ,, ,,
5. Gelatine (Collodion covered).

be done by kathode bombardment, is the chief factor against their common use. They would have to be between two plates of glass, but would make ideal heat and glare protectors, as most of the energy would be reflected and not absorbed, thus tending to keep the glass cool. Silver films act in nearly the same way, and can be chemically deposited. The colour of these is generally slightly blue, and they transmit fairly well between .33μ and .29μ. A silver film, however, makes a very agreeable "neutral" eye glass. The ultra-violet transmission would not be against it as the glass on which it would be mounted could easily be chosen so as to annul the effect. If a silver film transmits

10 per cent. of the light it will also transmit slightly less than 10 per cent. of the infra-red. It seems that these films would be preferable to glass for welding operations, &c., as they cut down light and heat at the same time. Most ordinary coloured glasses transmit well in the infra-red. Platinum films are another alternative; they can also be chemically produced, and, further, they can be burnt into the glass. They absorb more radiation than silver or gold, and reflect less at the shorter wave-lengths.

Another possible filter suggested by the work of Sir W. Crookes is found in black biotite or coloured mica. Thin films of this transmit a pleasant brownish green light, and have considerable absorption in the shorter infra-red and violet regions. Such films would also need protection in use.

Comparison with the Work of Sir W. Crookes[5].—Limits of transmission in the ultra-violet were obtained by Sir W. Crookes with a source giving continuous spectrum, and a spectrograph giving greater dispersion. With the iron arc and a smaller spectrograph the limit found tends to be at a shorter wave-length, and a definite meaning has now been assigned to it by quantitative absorption work.

In the visible region, Sir W. Crookes gives the luminosity of transmitted light as measured by a Chapman Jones Opacity Meter, whereas this paper gives a measure of the percentage energy transmitted over a certain range of wave-length, an entirely distinct result.

In the infra-red an "integrated" result was found by using the energy transmitted by black biotite. The transmission of this substance in the infra-red, as shown by results recently obtained with the apparatus described in this paper, varies considerably with the nature and origin of the specimen. A few results have already been given by Königsberger[17]. Generally it absorbs heavily in the visible and infra-red up to about 1.5μ or 2.8μ, depending on the variety tested. Maximum transmission is about 3.6μ. The exact form of the transmission curve is being investigated, but it will be seen that such a filter in conjunction with a Nernst glower (maximum energy at 1.2μ) will tend to give a fairly even radiation in the infra-red, although the results must not be expected to correspond exactly to those given in this paper, which express the percentage energy transmitted over a given range of wave-length.

It is hoped that these results may be of some use to those using or manufacturing anti-glare glasses. In conclusion, I desire to thank Prof. Fowler, F.R.S., for his valuable advice throughout the work, and Messrs. W. Bradfield and C. Gregory, B.A., for their assistance with some of the experiments.

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* Other literature on this subject :—

HALLAUER.—Ueber die Absorption Kurzwelligen Licht durch die mensliche Linse. Klin. Monatsbl. f. Augenheilk, Dec., 1909. vol. 47. p. 721.

SCHANZ AND STOCKHAUSEN.—Absorption Spectra of the Media of the Eye. Arch f. Ophth. 1908, vol. 69, pg. 1. Arch. f. Ophth. 1909, vol. 73, p. 184.

SORET.—Sur la transparence des milieux de l'œil pour les rayons ultra violets. Compt. Rend. 1879, vol. 88, p. 1,013.

Additional Note.

Since the writing of this paper, further work has been done on the ultra-violet spectrophotometer method. The transmissions of several biotite films in the visible spectrum were photographically measured with extreme care in this way, and the results were compared with those obtained with the Abney apparatus. It was found that perfect agreement existed between the results when transmission was assumed proportional to the ratio of sector openings, as is stated by Professor Howe.

Further tests have been made on the transmission of neutral glasses, in which it was found that practically all specimens showed a remarkable increase of transmission at the extreme violet; some specimens about 1 mm. thick transmit the ultra-violet between .3μ and .4μ practically as well as ordinary glass; only heavily coloured glasses show absorption worthy of note. At the red end of the visible spectrum also, transmission rapidly increases. These effects throw some light on the apparent discontinuities in some of the earlier curves.

L. C. M.

July, 1917.

DISCUSSION.

Mr. W. A. DIXEY said the paper was a rather difficult one to discuss without a little further consideration. It touched on points which were of very great interest to many present, and they all ought to be very grateful to Mr. Martin for putting the paper together. He was a little doubtful as to whether the curves which had been exhibited on the screen were particularly applicable to the everyday requirements of Opticians in supplying spectacle lenses; what was really required was an estimate of the definite physical effect of the several differing intensities of radiation. In the diagrams given one would have wished that the thickness, and also the surface, of the various glasses compared should have been similar in the different specimens, because those factors would affect the results enormously.

Mr. MARTIN said those details were given in the paper.

Mr. DIXEY, continuing, said he had been interested in Mr. Martin's allusions to possible eye-strain in using instruments. He appeared to attribute the strain to possible want of clearness of definition. A spectacle-man, or an Optician, however, perhaps saw the matter a little more clearly than a laboratory experimenter. It appeared, for instance, to him (Mr. Dixey) that it was mainly a question of accommodation. When using a binocular instrument the question of accommodation was complicated by that of convergence. One ought to realise that an exact correlation, whether normal or created by habit, existed between the two functions, and if that was not carefully guarded in the adjustment of the instrument, eye trouble was sure to follow; it might be called strain, confusion, nausea or any other name. In a monocular instrument the main thing to remember was that the accommodation could be strained in only one direction. He had likened it very often to a hinged door. If there was over-focus of the instrument, beyond the one point of the exact focus, and the rays were unduly converged, the accommodation of the eye could not follow; it was up against the jamb of the door. But if the instrument was under-focussed, so as to produce

diverging rays, then the accommodation would adapt itself to that under-focus, and if one continued to use an instrument like that, sight might be easy and exact, but eye-strain would result. He was aware that his criticism of the paper was very inadequate, but he did not like to say anything further upon the subject of the use of different glasses and their effects on the eye without studying the paper, which he should take an opportunity of doing as soon as it was printed.

Mr. T. SMITH wished to ask Mr. Martin whether he had stated in his paper the effects on the eye for which he was seeking protection—whether, for instance, he was seeking to exclude radiation which caused the crystalline lens to become opaque, or produced an injury to the retina, or resulted in some other damage to the eye. He would also like to ask whether the results which Mr. Martin had tabulated referred to the total loss of light in the different specimens, or whether he had subtracted the loss due to reflection. It would in any case, he thought, be useful to state what approximately were the refractive indices of the glasses, with the different wave-lengths, so that one effect might be separated from the other, or the two added together, whichever way he had put them round. Further, was it the case that in the first and third diagrams shown on the screen the results obtained in the visible spectrum were discontinuous from those for the ultra-violet and infra-red? From the brief inspection possible while the slides were shown he thought there were appreciable discontinuities.

Mr. HUGHES asked if Mr. Martin's experiments could be used for the standardisation of tints. As far as he knew, there was no definite standard. He was of opinion that tinted screens could be used a great deal more for terrestrial observations than they were.

Mr. H. R. FAIRBANKS thought the Society was much indebted to Mr. Martin for having brought forward this subject, and for the way in which he had plotted out the curves of the different glasses. If they set to work to investigate these curves in the way Mr. Martin had shown them, it would be very helpful in the direction of getting a clearer idea of the properties of coloured glasses. He sympathised with Mr. Dixey in his remark that the paper required careful study before it could be criticised in detail. There was one kind of glass in which he was intensely interested, to which he thought Mr. Martin had scarcely devoted so much attention as it deserved. He referred to signal green glass. He considered this a very valuable glass indeed, and its action in the ultra-violet was peculiar. All microscopists knew that signal green glass formed a strong basis for a monochromatic screen. There was another point on which he would like a little further explanation. Mr. Martin had spoken about the absorption band at the D line in the Crookes glass. He had not quite caught Mr. Martin's explanation, but he thought that that absorption band in the Crookes glass was one of the most valuable properties

of the glass. It would be found that the absorption band at the D line was in every piece of Crookes glass which was examined. Whether or not this was due to water conditions, he would not like to say, but he rather doubted it.

Mr. S. A. Willmott remarked that when one looked at the moon through a powerful telescope, there was a feeling of discomfort, whereas when observed with the naked eye there was no such feeling. In the former case the image was not any brighter on the eye, but it was very much larger. He did not know whether Mr. Martin could explain why this discomfort should be experienced on viewing the moon through a telescope.

Dr. R. Mullineux Walmsley said the historical work embodied in Mr. Martin's paper would be a great convenience to many who were not in a position to refer to the original authorities. Like other speakers, he felt it difficult to attempt to criticise in detail a paper so packed with information, especially as it had been read in abstract; it was, therefore, only possible for an auditor to deal with odd points which had occurred to him during the reading of the paper.

One such point which had struck him (Dr. Walmsley) was this. The members of the Optical Society had a great respect for the heterogeneity of glass, its composite nature, its grading, and its methods of manufacture. What they would like to know from Mr. Martin—though where so much work had already been done, involving very great labour, one hesitated to ask for more—was the effect of the colouring matter itself upon absorption. In order to get at that, it would be necessary to compare the absorption of the actual glass obtained from the same melting before the addition of the colouring matter with the absorption of the glass obtained after the colouring matter had been added. The prospect of having to do all this in detail was something awful, and he did not know whether some of the glass-manufacturing experts might not say it was impossible.

Then another quite minor suggestion. In the first slide shown by Mr. Martin, he had drawn attention to the maxima along the spectrum, and it occurred to him (Dr. Walmsley) that if a dotted line were drawn through these maxima it would make the slide much more effective.

Mr. P. F. Everitt desired to ask Mr. Martin a question with regard to the stained films. Many of the dyes used for staining films had a fluorescent action, and the question occurred whether pure stains or mixed stains were used for such purposes, and whether the order of the stains might not make quite a difference in the transmission spectrum.

Mr. Rheinberg said he would like to ask Mr. Martin whether the question of screens for comfort and for visual acuity, &c., had received consideration from a somewhat different standpoint, viz., the evolutionary

standpoint, as it seemed to him that it led directly to certain inferences. It might be taken for granted that the eye, by the process of evolution, was naturally adapted to ordinary light—sunlight or skylight—and, therefore, that the radiations in their normal proportions which occurred in such light would not be harmful, whereas any abnormal excess of any of these radiations, or the presence of other radiations, might *a priori* be taken to be likely to be more or less detrimental, and where they occurred in artificial lighting, &c., it would seem safer if the screen eliminated them.

Then again as regards screens for acuity of vision. The process of evolution had determined the sensitiveness of the eye for colours, the normal curve showing a maximum in the yellow or greenish yellow, and it might therefore be assumed straight away that this would be where the geatest visual acuity would be found. As owing to the well-known chromatic aberration of the eye, the different colours were not simultaneously in focus, the screen would be one which chiefly transmitted this region of the spectrum and cut down the other colours. He believed that in practice nearly all screens for visual acuity would be found to be yellow or greenish yellow.

Dr. WALTER ROSENHAIN regretted that having unfortunately arrived late he had not heard the whole paper. He was, however, very much interested in the part he had heard, and he thought the paper would prove valuable, as much for the methods described as for the results. He could not, of course, attempt to criticise the paper, but wished to mention two small points which, though not directly connected with the paper, nevertheless had reference to its subject. First of all there was the very wide subject of the choice of absorption glasses. He had had in 1914 the privilege of a voyage through the tropics and back again in connection with the British Association's visit to Australia, and, through the courtesy of Messrs. Chance, he had been able to try on that occasion two pairs of glasses made of Crookes glass, one very lightly and the other heavily tinted. He had never found any use for the heavily tinted pair, but the light pair had proved most extraordinarily comfortable—far more so than a pair of ordinary tinted glasses which he had also had with him for the sake of experiment. Unfortunately his eyes were distinctly sensitive to bright light, and he had found the light-tinted Crookes glasses extremely comfortable and perfectly neutral. He did not know how it was to be accounted for, but they certainly had produced the desired effect. Then with regard to the use of tinted glasses when observing with an instrument for a length of time, no doubt this would lead to considerable rest and comfort, and there was a very simple way of doing it. With very little practice it was, he thought, perfectly possible to use any optical instrument, with the exception, perhaps, of microscopes

employing very high power eye-pieces, with any pair of spectacles, and of course the glasses could be of any colour one preferred. In this way it was possible, in using a telescope, to secure the desired ease and comfort without any special attachment whatever.

The PRESIDENT said there was one small matter to which he wished to direct attention before calling upon Mr. Martin to reply. Mr. Willmott had pointed out the fact that the glare of the moon through a telescope produced discomfort, while no such effect was caused by looking at that luminary with the naked eye. In connection with that he might mention one of the most extraordinary, and still unsolved, of optical problems—that of Herschell. Within a short distance of Herschell's observatory was a village church and clock by which the time could be seen with the unaided eye in the daytime. At night-time, when so dark that the church itself was invisible, Herschell found that by the use of his reflector he could not only see the church, but the time indicated by the clock itself. Now it was of course an axiom in optics that the intrinsic brightness of a retinal image, no matter what optical aid was employed, could never exceed the intrinsic brightness of the retinal image produced by direct and unaided vision.

Mr. S. D. CHALMERS said that the problem in question had probably been solved by the observations of Patterson and Dudding on the visibility of small sources. The results and some considerations of the speaker were included in a discussion on "Visibility" at the Illuminating Engineering Society.*

AUTHOR'S REPLY.

Mr. MARTIN, in replying, said that, with regard to the definition in the field of view of a telescope, accommodation would enable the image to be focussed if the instrument were slightly out of adjustment, but the bad definition to which he had referred was chiefly due to residual chromatic aberration; this became very trying when observing, say, a small white flag for signalling purposes.

The chief effects of ultra-violet rays were discomfort, inflammation of the conjunctiva, and other phenomena, with opacity of the cornea in extreme cases. Infra-red rays were also liable to produce cataract with long-continued exposure.

Tinted glasses could be exactly standardised by obtaining the absorption curves, but difficulties might be experienced in exactly reproducing a glass as the tint was liable to vary somewhat in different meltings.

* The Illuminating Engineer, p. 210, No. 5, Vol. VIII., May, 1915.

Signal green was doubtless a valuable glass, and fuller details were given in the paper. With regard to the absorption band in the yellow, in Sir W. Crookes' spectacle glass, this was due to Didymium, which he had understood to be an unnecessary impurity.

Strain in observing the moon through a telescope was probably due to lack of adaptation. As far as retinal glare was concerned, the size of the source of light was of great importance. A small object, such as a condensed electric spark, could be looked at without discomfort, although it might give a very strong radiation. The same applied to a distant arc lamp. Thus the direct observation of the moon without a telescope would affect a very small area of the retina, and would not produce discomfort. When a telescope was used, an intense light was thrown on to a comparatively large area of the unadapted retina, and glare resulted. The observation was less trying in daylight when the eyes are adapted to stronger illumination.

He agreed with Dr. Walmsley that it would have been desirable and interesting to have given for each glass the absorption, with and without the colouring oxides, but the glasses tested were commercial samples of unknown composition. The *base* was generally a simple soda lime glass, the absorption of which has been thoroughly investigated. Such results were given in Hovestadt's " Jena Glass."

The testimony of Dr. Rosenhain as to the value of " Crookes " glass was extremely interesting, and doubtless the experience of such a skilled observer was a valuable indication of the utility of the glass. He did not think, however, from knowledge of particular cases, that such glasses were a panacea for relieving eye-strain and glare, although they might be extremely useful in special cases.

Mr. Everitt's remarks on celluloid filters were of great interest. He had had several examples of stained films sent him which were very efficient for their particular uses. He had no experience of the preparation of these, but at first sight it did not appear that the mere order of the stains should affect the absorption.

From personal experience in Gallipoli, he fully realised the strain in the use of a telescope for prolonged periods under intense illumination. This was a serious matter, and he hoped that the few results in the paper would lead to some investigation of the subject.

Good definition doubtless contributed greatly to comfort in vision, but the increase in acuity when using a relatively monochromatic glass as against a neutral glass was hardly sufficient to give any appreciably greater comfort. The increased contrast may be more satisfactory to the eye.

With regard to the President's remarks, it was of great interest to hear that some explanation had been offered of the curious effect noted by Herschell. The explanation of the visibility of stars from the bottom of a deep shaft was a somewhat similar problem.

Mr. T. SMITH: May I ask whether that is a fact?

The PRESIDENT: I may say I put that problem to the Astronomer Royal some time ago, and he would not commit himself.

Mr. Martin, continuing, said that although some light had been thrown on these questions, they did not appear to be fully explained.

In reply to Mr. T. Smith, the results given expressed the actual transmission of specimens after reflection and absorption. In calculations of the transmission of other thicknesses for technical purposes, it was sufficiently accurate to assume a mean value for the refractive index and consequent reflection.

There were two small regions at each end of the visible spectrum .40μ-.438μ, and .69μ-.76μ, in which it was difficult to obtain measures of absorption with his existing apparatus. In some of the earlier experiments these intervals were wider, and unfortunately the measurement at about .76μ with the infra-red apparatus was the most uncertain, as the galvanometer deflections were very small at this point. Some of the glasses tested had a somewhat rapidly changing transmission in these regions, especially green and neutral tints. Thus the curves did not always appear to join. The differing scales of wave-length must be remembered when looking at the results.

Note on Accuracy of Observation and Precision in Measurement.

By G. A. CARSE, M.A., D.Sc.

Read and discussed April 12th, 1917.

MEASUREMENTS are made by comparison with some standard. The fundamental standards are those of length, mass and time, and two systems are in general use—1, the foot-pound-second system or shortly the F.P.S. system, the units of length, mass and time being respectively the foot, pound and second, and 2, the centimetre-gramme-second system—the C.G.S. system, the units here being the centimetre, gramme and second. From these fundamental units certain other units termed Derived Units are obtained, *e.g.* unit of speed, &c.

The value of certain constants is obtained by computation after certain direct observations have been made, *e.g.*, one method of determining the refractive index of a prism is to measure an angle of the prism and the angle of minimum deviation, and then by calculation from a formula the refractive index is obtained; the refractive index is an *indirect* observation to obtain which two direct observations have been made.

Now any measurement is liable to error. One observer may be a better observer than another, and one measuring instrument may read to a greater degree of accuracy than another. If a yard stick is one inch too long, then each yard measured would be in error by one inch, and thus there would be introduced a *systematic* error which could be corrected or allowed for, provided its magnitude is known; moreover, this error will always have the same sign. On the other hand errors due to personal observation will vary in magnitude and sign from observer to observer, and from measurement to measurement in the case of one observer. They will occur sometimes as positive errors and sometimes as negative errors, sometimes large and sometimes small, and errors of this nature of varying magnitude and with a random distribution of signs are termed *accidental* errors. Systematic errors will always follow a definite law, and if this law is known can be allowed for or eliminated, whilst accidental errors will be of varying magnitudes and signs, and in general can only be allowed for approximately.

It is therefore our purpose to explain a few simple methods, which are easy of application, by which accidental errors can be allowed for, it being presumed that systematic errors have been eliminated or corrected for. A full discussion would of course involve the theory of probabilities which will not be touched on here.

Since accidental errors occur sometimes as positive errors and sometimes as negative ones, and of varying magnitude, it is clear that we shall in general arrive at a much closer approximation to the real or true value by taking the arithmetic mean of a series of observations of the same measurement taken under the same conditions; for then in the final result the positive and negative errors will tend to cancel or neutralise each other, and the mean will in general be a better determination than any single observation, and will be a closer approximation to the true value the greater the number of observations made.

Possible Error of an Observation.

(1) A DIRECT OBSERVATION.—By keeping all the conditions the same and repeating an observation a number of times we can, by taking the mean of the result, arrive at an estimate of the possible error in an observation; *e.g.*, if the values of a measurement repeated five times successively be 7.40, 7.35, 7.40, 7.30, 7.45, then the mean of these, viz. 7.38, may be taken as the true value, and the deviations of the above values from this mean may be considered as the errors. In the series given above the greatest deviation from the mean is .08, and this may be taken as an approximation to the *possible error* or *error limit* of an observation. The possibility of an error of .08 in any *single* observation may be indicated by writing the observation in question, viz., say 7.35 as $7.35 \pm .08$, and the error is here expressed *absolutely* as it is termed. If it is expressed as a fraction of the mean, then the *fractional* error is $\frac{8}{738}$, or $\frac{1}{90}$ say, or as a percentage of the mean—the *percentage* error is 1.1 per cent.

It may not always be possible to repeat an observation under exactly the same conditions, and various methods, which will not be dealt with here, have been devised to overcome the difficulty in such a case.

(2) AN INDIRECT OBSERVATION.—(*a*) An obvious method analogous to that of a direct observation is to make a series of determinations of the quantity under consideration, *e.g.*, focal length of a lens. These observations in this example would be calculated in this instance, let us say, from direct observations of the distances of object and image from the lens and the greatest deviation from the mean of the values of the focal length is then obtained. This is the possible error of a single determination of focal length, and it has been experimentally determined.

(*b*) The error can also be determined by calculation when the possible errors in the components (*i.e.*, the direct observations necessary to obtain the final result) have been determined as shown in (1).

We now proceed to give some examples of the calculation of expressions for the possible errors of indirect observations, when the relations between the components are of a simple form.

Expressions for Possible Errors of Indirect Observation.

x and y are the magnitudes determined, and a is the constant obtained by calculation.

(1) Relation of the form $x\,y = a$

If a = possible error in x

β = " " y

λ = " " a

then

$$\lambda = (x + a)(y + \beta) - x\,y$$
$$= \beta\,x + a\,y \text{ (neglecting quantities of the second order, viz. } a\,\beta)$$
$$= x\,y\left(\frac{\beta}{y} + \frac{a}{x}\right)$$

hence

$$\frac{\lambda}{a} = \frac{\beta}{y} + \frac{a}{x}$$ i.e. fractional error in a = sum of fractional errors in x and y.

(2) Relation of the form $y = ax$ or $y/x = a$

a, β, λ as before,

then

$$\lambda = \frac{y + \beta}{x - a} - \frac{y}{x} = \frac{y + \beta}{x\left(1 - \frac{a}{x}\right)} - \frac{y}{x} = \frac{\beta}{x} + \frac{y}{x^2}a$$

neglecting quantities of the second order (i.e. a^2 and $a\,\beta$)

$$= \frac{y}{x}\left(\frac{a}{x} + \frac{\beta}{y}\right)$$

hence

$$\frac{\lambda}{a} = \frac{a}{x} + \frac{\beta}{y}$$ i.e. fractional error in a = sum of fractional errors in x and y.

(3) Relation of the form $y^2/x = a$.

$$\lambda = \frac{(y + \beta)^2}{x - a} - \frac{y^2}{x} = \frac{(y + \beta)^2}{x\left(1 - \frac{a}{x}\right)} - \frac{y^2}{x} = 2\beta\frac{y}{x} + \frac{y^2}{x^2}a, \text{ approximately}$$

$$= \frac{y^2}{x}\left(\frac{a}{x} + \frac{2\beta}{y}\right)$$

also $\dfrac{\lambda}{a} = \dfrac{a}{x} + \dfrac{2\beta}{y}$

We see that since the *square* of y and only the first power of x occurs in the expression, the effect on the result of an error in y is twice that of an error in x.

Generally if $a = x^p y^q$ p and q having any values positive or negative, integral or fractional

$$\frac{\lambda}{a} = p\frac{\alpha}{x} + q\frac{\beta}{y},$$

the signs of α and β being chosen so as to make the resultant error a maximum.

DISCUSSION ON DR. CARSE'S PAPER.

Mr. EVERITT wished to draw the author's attention to a very frequent source of error. It was a source of error which was often overlooked, and it had not been dealt with in the present paper. He referred to errors due to personal equation.

Dr. CARSE pointed out that the whole paper had dealt with accidental errors, *i.e.*, errors which were not systematic, or at least which could not be included under systematic errors due to personal observation. One observer might be a better observer than another, and it was well known that in the case of a single observer his determination from observation to observation of the same thing varied. That was of course the fundamental basis of the whole discussion.

Mr. EVERITT assented, but said that according to that Dr. Carse treated these as accidental errors, there being no connection between one observation and the next. But it was well known that errors due to personal equation had a very high connection with each other.

DR. CARSE: Then it becomes a constant error.

MR. EVERITT: But that does not persist after a certain length of time. It is neither accidental nor constant, but something in the nature of each.

THE PRESIDENT: Can you give us anything in the shape of a concrete example, Mr. Everitt?

MR. EVERITT said the simplest case of all was to take a line of given length, have it printed on paper, and bisect it with a pencil, later on putting

a scale on it and seeing what the error is. In that case it would be found that the error in any one observation was correlated to errors in the succeeding observations. The correlation gradually decreased in intensity, but was quite persistent up to about the tenth observation. That was the simplest case he knew of, and the one most easily investigated. Personal equation was a very important factor, especially in measuring. It was of great importance where a mark on a scale was set between two wires in an eyepiece—a very common type of measuring instrument.

MR. R. S. WHIPPLE thought that Dr. Carse had dealt with a very important matter. He believed that the most successful investigator was the man who could judge at once the percentage of accuracy required in any observation. He had had the privilege of spending a day with Sir Ernest Rutherford when he was at the Macgill University, where he had five experiments going on at the same time. First of all they went up to the roof, and the Professor took observations there; then they went down to the basement and he took further observations there, and so on throughout the building. When discussing the details of one of these observations, he remarked to the Professor "Don't you think there is an error there?" and the reply was, "It is only a two per cent. error and I can only hope for an accuracy of ten per cent. in this experiment." He remembered once helping a distinguished magnetician with some magnetic dip observations. The observations were reduced to three places of decimals, viz. to thousandths of a minute of arc when they could not be relied upon to a minute. This was a striking example of an investigator working without judging the importance of his errors. It was the same in mechanical matters; a man was very apt to strain at accuracy in non-essentials, and not to know where accuracy was required. He looked forward to the publication of Dr. Carse's paper with great interest, as he believed it would help observers, who perhaps were not highly trained, to determine whether an error was important or not.

THE PRESIDENT said he remembered, in connection with the subject introduced by Dr. Carse, a famous story told by Professor Perry. There was a certain book on Astronomy, or perhaps on Physical Geography, in which the distance of the sun from the earth was set out very clearly as being 95,463,248 miles; and the Professor enquired why the distance had not been given to the nearest furlong. The explanation was quite simple. The figures had been taken from a French book, in which, of course, the distance had been given in a round number of metres; but in the translation this round number had been very conscientiously converted into miles. Another point which he hoped Dr. Carse, in his detailed paper, would deal with effectively was the waste of time in the use of logarithms. Many of

his friends who had obtained results which were probably reliable to no more than the second or third figure, would insist on using six, seven, or eight figure logs; and he believed they would use twelve or fifteen if they could get them.

AUTHOR'S REPLY.

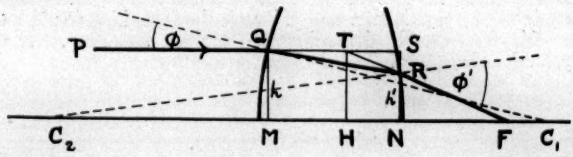
DR. CARSE, in reply, said he had not at first quite understood Mr. Everitt's point about the gentleman who divided the line into intervals. It seemed that after about ten observations he appeared to get the result fairly correct. That would be a systematic error, and, as he had pointed out, the law of systematic error was not necessarily a linear law. No matter how complex the calculation, it was possible to correct if one knew the law of the error introduced by bisecting the line, by trial. Taking Mr. Everitt's example, his first ten observations would require corrections of systematic error; and in that case of course the best thing would be to begin with the eleventh, discarding the first ten altogether. It was partly, he thought, a psychological effect.

A Simple Proof of the Expression for the Focal Power of a Thick Lens.

By CHARLES COCHRANE, M.A., B.Sc.

Presented 12th April, 1917.

THE derivation of the well-known expression for the power of a thick lens for axial rays is usually treated as a corollary in the general analytical discussion of the theory of thick lenses and systems of lenses. An exceedingly simple and direct proof, however, can be obtained, from purely geometrical considerations, by the use of a method of exposition frequently adopted in the case of the thin lens.



P Q R F, represents the path of a ray which is incident parallel to the axis of the lens. C_1 and C_2 are the centres of curvature of the two faces of the lens, which have radii of lengths r_1 and r_2 respectively. P Q and F R produced intersect in T. T H, drawn perpendicular to thea xis, represents, therefore, the principal plane of the image space, and H F is equal to f, the equivalent focal length of the lens. The angles of incidence and emergence are denoted by ø and ø′ respectively, and the corresponding angles C_1 Q R and C R Q within the lens by θ and θ′. Q M = h, R N = h', and N F = a.

For axial rays the angles are small, and we have numerically

$$\mu\theta = \phi = \frac{h}{r_1}, \quad \mu\theta' = \phi' = \frac{h'}{a} + \frac{h'}{r_2}, \text{ and } \theta + \theta' = \frac{h}{r_1} + \frac{h'}{r_2},$$

where μ is the index of refraction of the material of the lens.

$$\phi + \phi' = \mu(\theta + \theta')$$

$$\therefore \frac{h}{r_1} + \frac{h'}{a} + \frac{h'}{r_2} = \mu\left(\frac{h}{r_1} + \frac{h'}{r_2}\right),$$

$$\text{or} \quad \frac{h'}{a} = (\mu - 1)\left(\frac{h}{r_1} + \frac{h'}{r_2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Also

$$h - h' = \text{S R} = \text{Q S}(\phi - \theta) = t(\mu - 1)\frac{h}{\mu r_1},$$

where t is the thickness of the lens.

$$\therefore \; h' = h\left\{1 - \frac{(\mu - 1)t}{\mu r_1}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and

$$\frac{\text{T H}}{\text{H F}} = \frac{\text{R N}}{\text{N F}}, \quad \text{or} \quad \frac{h}{f} = \frac{h'}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Substituting from (2) and (3) in (1), and dividing by h, we get

$$\frac{1}{f} = (\mu - 1)\left\{\frac{1}{r_1} + \frac{1}{r_2} - \frac{(\mu - 1)t}{\mu r_1 r_2}\right\}.$$

First Approximation Relations in Centred Optical Systems.

By S. D. CHALMERS, M.A.

Presented 12th April, 1917.

THE object of this paper is to present a series of relations for centred optical systems which are analogous to the ordinary 1st order relations. (*)

As the methods of obtaining these results are strictly analogous to those employed in obtaining the Gauss relations, it will be convenient to develop and extend the Gauss relations and explain the modifications necessary to obtain the new relations.

REFRACTION AT A SINGLE SPHERICAL SURFACE.—Let P_1B_1 (Fig. I.) be a spherical surface separating two media of refractive indices n_0 and n_2. Let C_1 be the centre of curvature and P_1 any point on the surface. Let the ray which was travelling in the direction P_1A_0 in the medium n_0 be deviated to P_1A_2 in the medium n_2.

Denoting the angles $C_1P_1A_0$ by i_1 and $C_1P_1A_2$ by j_1, the law of refraction gives

$$n_0 \sin i_1 = n_2 \sin j_1.$$

In the triangle $C_1P_1A_0$ we have

$$\frac{\sin i}{\sin C_1} = \frac{C_1A_0}{P_1A_0}$$

so that

$$n_0 \frac{C_1A_0}{P_1A_0} = \frac{n_0 \sin i_1}{\sin C_1} = \frac{n_2 \sin j_1}{\sin C_1} = n_2 \frac{C_1A_2}{P_1A_2}$$

If the perpendicular $C_1Q_0 = p_0$ be dropped from C_1 on P_1A_0 we have

$$P_1A_0 = (C_1P_1^2 - p_0^2)^{\frac{1}{2}} + (C_1A_0^2 - p_0^2)^{\frac{1}{2}}$$

and this expression can be expanded in powers of p_0^2 so that

$$P_1A_0 = C_1P_1 + C_1A_0 - \tfrac{1}{2}p_0^2\left(\frac{1}{C_1P_1} + \frac{1}{C_1A_0}\right) + \text{terms in } p_0^4, \text{ etc.}$$

Similarly, dropping a perpendicular p_2 on P_1A_2

$$P_1A_2 = C_1P_1 + C_1A_2 - \tfrac{1}{2}p_2^2\left(\frac{1}{C_1P_1} + \frac{1}{C_1A_2}\right) + \text{terms in } p_1^4, \text{ etc.}$$

(*) These results were first pointed out by the author in a course of lectures at the Cavendish Laboratory, Cambridge, in 1910.

If the terms in p_0^2 and p_2^2 can be ignored, all rays proceeding to A_0 will be deviated to A_2 whatever be the position of P_1, and A_2 will be the image of A_0.

To this degree of approximation we have

$$n_0 \frac{C_1 A_0}{C_1 A_0 + r_1} = n_2 \frac{C_1 A_2}{C_1 A_2 + r_1} \quad \ldots\ldots\ldots \text{I}$$

Inverting and dividing by r_1 we have

$$\frac{C_1 A_0 + r_1}{n_0 C_1 A_0 . r_1} = \frac{C_1 A_2 + r_1}{n_2 C_1 A_2 . r_1}$$

or

$$\frac{1}{n_0 C_1 A_0} + \frac{1}{n_0 r_1} = \frac{1}{n_2 C_1 A_2} + \frac{1}{n_2 r_1} \quad \ldots\ldots\ldots \text{I}a$$

The relation I may also be written

$$n_0 \frac{C_1 A_0}{B_1 A_0} = n_2 \frac{C_1 A_2}{B_1 A_2}$$

so that

$$n_0 \frac{B_1 A_0 - r_1}{B_1 A_0 . r_1} = n_2 \frac{B_1 A_2 - r_1}{B_1 A_2 . r_1}$$

or

$$\frac{n_0}{B_1 A_0} + \frac{n_0}{r_1} = \frac{n_2}{B_1 A_2} + \frac{n_2}{r_1} \quad \ldots\ldots\ldots \text{I}b$$

which is the ordinary conjugate focus relation for a single surface.

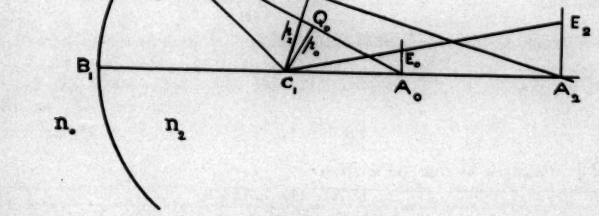


FIG. 1.

IMAGE OF A SMALL OBJECT AT RIGHT ANGLES TO THE AXIS.—If a small object A_0E_0 be considered, the ray proceeding through C_1 to the point E_0 will meet the plane (at right angles to the axis) which passes through A_2 in the point E_2, so that

$$\frac{A_2 E_2}{A_0 E_0} = \frac{C_1 A_2}{C_1 A_0} \quad \ldots\ldots\ldots\ldots \text{II}$$

and the point E_2 will be a point in the image patch of E_0, and may be regarded as the ideal image point;

$$\frac{A_2 E_2}{A_0 E_0}$$

is the linear magnification produced by the surface and will be denoted by m_1. It will, however, be convenient to use the value

$$\frac{n_2}{n_0}\frac{A_2 E_2}{A_0 E_0}$$

which will be denoted by M_1; so that

$$M_1 = \frac{n_2 C_1 A_2}{n_0 C_1 A_0} \quad \ldots\ldots\ldots\ldots \text{II}a$$

If we denote

$$\frac{1}{n_0 C_1 A_0} \text{ by } S_1 \text{ and } \frac{1}{n_2 C_1 A_2} \text{ by } T_1$$

the relation I*a* may be written

$$T_1 - S_1 = \left(\frac{1}{n_0} - \frac{1}{n_2}\right)\frac{1}{r_1}$$

and the last term, being independent of A_2 or A_0, is a property of the surface only, and may be denoted by P_1.

Thus we have

$$\left.\begin{aligned} T_1 - S_1 &= P_1 \\ \text{and} \qquad M_1 &= \frac{S_1}{T_1} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \text{III}a$$

from which we may deduce the relations,

$$\left.\begin{aligned} M_1 &= 1 - \frac{P_1}{T_1} \\ \frac{1}{M_1} &= 1 + \frac{P_1}{S_1} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \text{IV}a$$

The equation II may be written

$$m_1 = \frac{C_1 A_2}{C_1 A_0} = \frac{B_1 A_2}{n_2} \Big/ \frac{B_1 A_0}{n_0} \quad \ldots\ldots\ldots\ldots \text{II}b$$

where m_1 is the linear magnification, and denoting

$$\frac{n_2}{B_1A_2} \text{ by } V_1 \qquad \frac{n_0}{B_1A_0} \text{ by } U_1$$

we have

$$V_1 - U_1 = (n_2 - n_0)\frac{1}{r_1}$$

and the second term may be denoted by F_1, since it is independent of A_0, and is called the power of the surface.

Thus

$$V_1 - U_1 = F_1$$

and from II*b*

$$\frac{U_1}{V_1} = m_1 \qquad \text{III}b$$

From these relations we may deduce

$$\left.\begin{aligned} m_1 &= 1 - \frac{F_1}{V_1} \\ \frac{1}{m_1} &= 1 + \frac{F_1}{U_1} \end{aligned}\right\} \qquad \text{IV}b$$

It will be seen that the relations (*a*) are strictly analogous to the relations (*b*) which are the normal first order relations for a single surface.

In the equations (*b*) the distances B_1A_2, etc., occur divided by the refractive indices and the quantities

$$\frac{B_1A_0}{n_0}, \quad \frac{B_1A_2}{n_2},$$

etc. are termed "reduced" distances, and denoted by u_1 and v corresponding to their reciprocals U_1, V_1, etc.

In the equations (*a*) the distances C_1A_0, etc. occur, multiplied by the refractive indices, and the quantities $n_0 C_1A_0$, $n_2 C_1A_2$, etc., are termed optical distances, and denoted by s_1 and t_1 corresponding to their reciprocals S_1, T_1, etc.

LENSES.—We will find it convenient to denote the media by suffixes 0, 2, 4, etc., and the surfaces by 1, 3, 5, etc.

CASE OF THIN LENSES—In which the differences between B_1A_2 and B_3A_2, etc., can be neglected.

Let F_1 and F_3 be the powers of the surfaces, then we have

$$\begin{aligned} F_1 &= V_1 - U_1 \\ F_3 &= V_3 - U_3 \end{aligned}$$

where the suffix 3 refers to the second surface.

Since the separation may be neglected, $V_1 = U_3$

$$F_1 + F_2 = V_3 - U_1 \quad \ldots \quad V b$$

If $U_1 = o$, *i.e.*, the object is at infinity,
$V_3 = F_1 + F_3$, and is the focal power of the thin lens. In this case

$$v_3 = \frac{1}{V_3}$$

is the distance from the lens of the image of a distant object, and is called the focal length. Denoting the power of the lens by $F_{1,3}$ and the focal length by $f_{1,3}$ we have

$$\frac{1}{f_{1,3}} = F_{1,3} = F_1 + F_3 \quad \ldots \quad VI b$$

If the object be at infinity the size of the image depends on the angle subtended by the object. If the object extend from the axis to subtend an angle θ_0 at the first surface, the size of the image is $f_{1,3} \tan \theta_0$ . . VII *b*

THICK LENSES.—When the surfaces are separated we define the equivalent focal length and power as the focal length and power of the thin lens, which would give the same image of the same distant object.

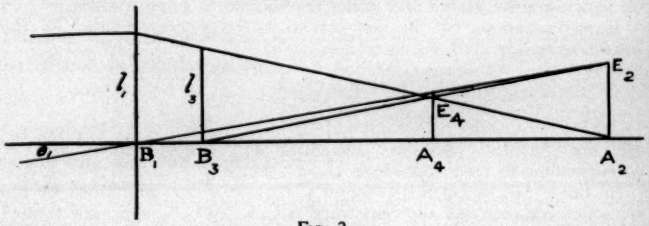


FIG. 2.

EQUIVALENT FOCAL POWER OF A THICK LENS.—Let F_1 and F_3 be the powers, B_1B_3 the separation of the two surfaces, and let B_1B_3/n_2 be denoted by d_2. Consider a distant object subtending an angle θ_0 at the first surface. Let the first image be formed at A_2E_2 and the second image at A_4E_4

$$A_2 E_2 = \frac{B_1 A_2}{n_2} \tan \theta_0 = f_1 \tan \theta_0$$

where

$$f_1 = \frac{1}{F_1}$$

$$A_4 E_4 = m_3 A_2 E_2 = m_3 f_1 \tan \theta_0$$

The equivalent focal length of the combination is $m_3 f_1$, and the equivalent focal power $= \frac{1}{m_3} F_1$

But $$\frac{1}{m_3} = 1 + u_3 F_3 \text{ where } u_3 = \frac{1}{U_3} = \frac{B_3 A_2}{n_2}$$

also

$$B_1 A_2 = B_1 B_3 + B_3 A_2$$

so that

$$\frac{B_1 A_2}{n_2} = \frac{B_1 B_3}{n_2} + \frac{B_3 A_2}{n_2}$$

that is

$$u_3 = f_1 - d_2 \quad \ldots\ldots\ldots\ldots \text{VIII}b$$

and the equivalent focal power $F_{1.3}$ of the lens is

$$F_{1.3} = F_1 + F_3 \{F_1 (f_1 - d_2)\}$$

$$F_{1.3} = F_1 + F_3 \left(\frac{f_1 - d_2}{f_1}\right) = F_1 + F_3 (1 - d_2 F_1) \quad \ldots \text{IX}b$$

INTERPRETATION.—Let a ray parallel to and near the axis meet the first surface at a distance l_1 from the axis and the next surface at a distance l_3 from the axis

$$\frac{l_3}{l_1} = \frac{B_3 A_2}{B_1 A_2} = \frac{f_1 - d_2}{f_1} = (1 - d_2 F_1) \quad \ldots\ldots \text{X}b$$

so that

$$F_{1.3} = F_1 + \frac{l_3}{l_1} F_3$$

and

$$v_3 = m_3 u_3 = \frac{u_3}{f_1} m_3 f_1 = \frac{l_3}{l_1} \cdot \frac{1}{F_{1.3}} \quad \ldots\ldots \text{XI}b$$

LENS SYSTEMS.—If a third surface be considered

$$F_{1.3.5} = m_5 F_{1.3} = (1 + u_5 F_5) F_{1.3}$$

$$= F_{1.3} + (u_5 F_{1.3}) F_5 \quad \ldots\ldots \text{XII}b$$

but

$$\frac{u_5}{v_5} = \frac{l_5}{l_3}$$

$$v_3 F_{1.3} = \frac{l_3}{l_1}$$

$$\therefore \quad u_5 F_{1.3} = \frac{l_5}{l_1}$$

also

$$\frac{l_5}{l_3} = \frac{v_3 - d_4}{v_3} = 1 - \frac{d_4}{v_3}$$

$$\therefore \quad \frac{l_5}{l_1} = \frac{l_3}{l_1} - d_4 \frac{l_3}{l_1 v_3} = \frac{l_3}{l_1} - d_4 \mathrm{F}_{1\cdot3} \quad \ldots\ldots \text{XIII}b$$

and

$$\mathrm{F}_{1\cdot3\cdot5} = \mathrm{F}_{1\cdot3} + \frac{l_5}{l_1}\mathrm{F}_5$$

and the result may be extended to any number of surfaces.

Returning to the original equations, we can obtain the effect of the lenses for any object.

In the case of an object not at infinity

$$m_1 \times m_3 = (1 - v_1 \mathrm{F}_1)\, m_3$$

where $v_1 = d_2 + u_3$,

so that

$$\begin{aligned} m_1 m_3 &= (1 - d_2 \mathrm{F}_1)\, m_3 - u_3 \mathrm{F}_1 m_3 \\ &= (1 - d_2 \mathrm{F}_1)(1 - v_3 \mathrm{F}_3) - v_3 \mathrm{F}_1 \\ &= (1 - d_2 \mathrm{F}_1) - v_3 \{\mathrm{F}_1 + (1 - d_2 \mathrm{F}_1)\mathrm{F}_3\} \\ &= \frac{l_3}{l_1} - v_3 \mathrm{F}_{1\cdot3} \end{aligned}$$

This result may be extended to any number of surfaces, and

$$m = \frac{l}{l_1} - v\,\mathrm{F} \quad \ldots\ldots\ldots\ldots \text{XIV}b$$

where the quantities without suffixes refer to the complete system.

In a similar way

$$\frac{1}{m} = \frac{l'}{l_1} + u\,\mathrm{F}$$

where l' is calculated for the reversed system.

These results, being universally true, must hold good when m is given special values.

Let $m = 0$, then

$$v\,\mathrm{F} = \frac{l}{l_1}$$

or the back focussing distance $n\,v$ for a distant object is given by

$$v = \frac{l}{l_1}\frac{1}{\mathrm{F}}.$$

Let $m = 1$, then

$$1 = \frac{l}{l_1} - v_p \mathrm{F}$$

where v_r is the reduced distance from the last surface of the plane of the image which is the same size as the object, *i.e.*, v_r is the reduced distance of the Principal Plane from the last surface.

The general equation then becomes

$$m = 1 - (v_r - v)\,\mathrm{F} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{XV}b$$

i.e., the same form as for a single lens, if $n\,v$ be measured from the Principal Point instead of the lens surface.

Similar considerations give the relation

$$\frac{1}{m} = 1 + (u - u_0)\,\mathrm{F}$$

where u_0 is the distance of the other Principal Plane from the first surface.

These two relations are algebraically equivalent to the relations $\mathrm{F} = \mathrm{V} - \mathrm{U}$ and $m = \mathrm{U}/\mathrm{V}$ where V and U are obtained from distances measured from the Principal Planes instead of the surfaces.

The consideration of all problems is much simplified if diagrams are drawn using the "reduced" distances

$$\frac{\mathrm{B_1\,A_2}}{n_2}, \quad \frac{\mathrm{B_1\,B_2}}{n_2}, \quad \frac{\mathrm{B_3\,A_2}}{n_2},$$

etc., instead of the real distances.

If this be done, rays through the vertex of any surface may be drawn as if they were not deviated at the surface, so that all image constructions are simplified. All image distances will then be expressed as reduced distances, and u_0, v_r, etc., will be reduced distances.

Special Relations.—We draw a ray from the object $\mathrm{A_0}$ to a point $\mathrm{Q_1}$ on the principal plane, and at a distance $\mathrm{Q_1\,Q}$ from the axis; this ray will emerge from the point $\mathrm{P_1}$, so that $\mathrm{P_1P} = \mathrm{Q_1\,Q}$, and meet the axis in A.

We have

$$m = \frac{\mathrm{P\,A}/n_2}{\mathrm{Q\,A_0}/n_0} = \frac{n_0\,\mathrm{Q_1\,Q}}{\mathrm{Q\,A_0}} \Big/ \frac{n_2\mathrm{P_1\,P}}{\mathrm{P\,A}} = \frac{n_0 \tan \mathrm{A_0}}{n_2 \tan \mathrm{A}} \text{ or } \frac{n_0\,\mathrm{A_0}}{n_2\,\mathrm{A}} \quad . \quad . \quad \text{XVI}b$$

to this approximation.

Exactly analogous relations may be worked out using the relations a instead of b.

We denote the reciprocals of $\mathrm{T_1}$ and $\mathrm{S_1}$ by t_1 and s_1, the value of $\mathrm{C_1C_3}\ n_2$ by a_2.

The special case of a thin lens (p. 125) is replaced by a concentric lens, in which case $\mathrm{P_{1.3}} = \mathrm{P_1} + \mathrm{P_3}$.

In the general case

$$s_3 = t_1 - a_2 \quad \ldots \quad \text{VIII}a$$

$$P_{1.3} = P_1 + P_3 (1 - a_2 P_1) \quad \ldots \quad \text{IX}a$$

The interpretation of $1 - a_2 P_1$ is obtained by considering the perpendiculars drawn from C_1, C_3, etc., on a ray initially parallel to the axis, denoting by $p_0 p_2$ the perpendiculars, as in Section I. We have $n_0 p_0 = n_2 p_2$, so that each of these is characteristic of the centre of curvature C. We denote this quality by q_1.

Then

$$\frac{q_3}{q_1} = \frac{C_3 A_2}{C_1 A_2}$$

that is

$$\frac{q_3}{q_1} = \frac{C_1 A_2 - C_1 C_3}{C_1 A_2} = \frac{\dfrac{1}{P_1} - a_2}{\dfrac{1}{P_1}} = (1 - a_2 P_1) \quad \ldots \quad \text{X}a$$

and

$$P_{1.3} = P_1 + \frac{q_3}{q_1} P_3$$

and

$$t_3 = \frac{q_3}{q_1} \frac{1}{P_{1.3}} \quad \ldots \quad \text{XI}a$$

Considering a third surface we have as before

$$P_{1.3.5} = P_{1.3} + \frac{q_5}{q_1} P_5 \quad \ldots \quad \text{XII}a$$

$$\frac{q_5}{q_1} = \frac{q_3}{q_1} - a_4 P_{1.3} \quad \ldots \quad \text{XIII}a$$

The equations

$$M = \frac{q}{q_1} - t P$$

and

$$\frac{1}{M} = \frac{q'}{q_1} + s P \quad \ldots \quad \text{XV}a$$

may be derived exactly as the corresponding equation XV*b*

The special case where M=1 leads to the equations

$$M = 1 - (t_N - t) F$$

and

$$\frac{1}{M} = 1 + (s_M - s) F$$

where the points N and M are such that

$$n_2 m = n_0$$

If N and M denote the angles with the axis made by a ray passing through N and M we have

$$m = \frac{n_0 M}{n_2 N} = \frac{n_0}{n_2}$$

so that M=N and the points M and N are the Nodal Points of the system.
(Note : C_1,C_3, etc., are the Nodal Points of the single surfaces.)

Optical Distances.—Results analogous to those relating to reduced distances may be obtained if all dimensions are multiplied by the refractive indices of the media in which the distances are measured, the centres of curvature being kept constant.

Thus, on passing through a surface of radius r_1 the ray is drawn to a point on the surface of radius $r_1 n_0$ and transferred to the corresponding point on the surface of radius $r_1 n_2$, the point of crossing, the central plane being kept constant.

The next central plane is drawn at a distance $a_2 = C_1 C_3 n_2$, and the ray produced to meet this plane and the next surface.

Thus all the relations which have been established for optical systems in relation to the Principal Planes can be transferred to Nodal Points by a suitable modification of the elements used.

To Find the Apparent Distance of a Point.

In the figure (Fig. 3) we draw the consecutive refracting surfaces P_1B_1, P_3B_3, etc., not at their true distances apart, but at their "reduced" distances, and all rays are drawn as if their focussing distances were reduced by dividing by the refractive index of the medium in which they are measured.

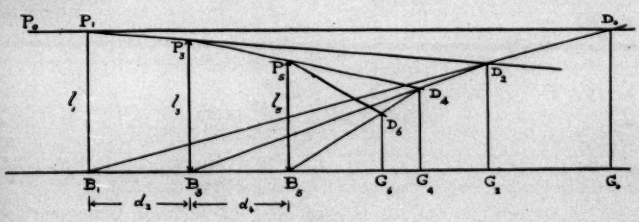


Fig. 3.

Thus, any two points conjugate with respect to a surface will lie on a straight line through the vertex of the surface.

Let the ray P_0P_1, parallel to the axis B_1B_3 meet consecutive surfaces

in P_1, P_3, P_5, etc., and any plane (6) in the medium (6) in D_6, and let the lines B_5D_6 and P_3P_5 meet in D, and the lines P_1P_3 and P_3D_4 in D_2.

Then the two rays which actually meet in D_6 would meet in D_4 in medium (4) or D_2 in medium (2) or D_0 in medium (0), as that D_4 is conjugate to D_6 with regard to the surface (5), and to D_2 with regard to the surface (3); while D_0 is the conjugate point in the medium (0).

Let D_6G_6, D_4G_4, D_2G_2 and D_0G_0 be drawn at right angles to the axis.

From the figure

$$\frac{B_5\,G_6}{D_6\,G_6} = \frac{B_5\,G_4}{D_4\,G_4} \quad \text{XVII}$$

or

$$\frac{B_5\,G_6}{l_5\,D_6\,G_6} = \frac{B_5\,G_4}{l_5\,D_4\,G_4} \quad \text{XVII}b$$

But since $P_3P_5D_4$ is a straight line

$$\frac{P_3\,D_4}{l_3\,D_4\,G_4} = \frac{P_3\,P_5}{l_3\,l_5} + \frac{P_5\,D_4}{l_5\,D_4\,G_4} \quad \text{XVIII}b$$

or since P_3 and P_5 are near the axis

$$\frac{B_3\,G_4}{l_3\,D_4\,G_4} = \frac{d_4}{l_3\,l_5} + \frac{B_5\,G_4}{l\;B_4\,G_4} \quad \text{XIX}b$$

In a similar manner

$$\frac{B_3\,G_4}{l_3\,D_4\,G_4} = \frac{B_3\,G_2}{l_3\,D_2\,G_2}$$

and

$$\frac{B_1\,G_2}{l_1\,D_2\,G_2} = \frac{d_2}{l_1\,l_3} + \frac{B_3\,G_2}{l_3\,D_2\,G_2}$$

so that

$$\frac{B_1\,G_2}{l_1\,D_2\,G_2} = \frac{d_2}{l_1\,l_3} + \frac{B_3\,G_2}{l_3\,D_2\,G_2} = \frac{d_2}{l_1\,l_3} + \frac{B_3\,G_4}{l_3\,D_4\,G_2}$$

$$= \frac{d_2}{l_1\,l_3} + \frac{d_4}{l_3\,l_5} + \frac{B_5\,G_4}{l_5\,D_4\,G_4}$$

$$= \frac{d_2}{l_1\,l_3} + \frac{d_4}{l_3\,l_5} + \frac{B_5\,G_6}{l_5\,D_6\,G_6} \quad \text{XX}b$$

So that if we take the various "reduced" distances d_2, d_4, B_5G_6, and divide each by the product of the intercepts $l_1\,l_3$, etc., at its two ends, the sum of these quantities gives the value of the corresponding quantity for the distance (B_1G_2) of the plane conjugate to G_6D_6 from the same surface B_1.

In the special case where the conjugate plane in medium (0) is required we have

$$\frac{d}{l_1 l_1} = \sum_2^4 \frac{d_2}{l_1 l_3} + \frac{u_6}{l_5 l_6} \quad \ldots\ldots\ldots \text{XXI}b$$

where u is written for B_5G_6 and l_6 for D_5G_6, and d is the distance of the plane conjugate to G_6D_6 and measured from the first surface.

By the relations above

$$l_6 = l_5 - l_1 u_6 F_{1.3.5}$$

To find the Principal Plane at Incidence

$$\frac{d}{l_1 l_1} = \sum \frac{d_2}{l_1 l_3} + \left(\frac{l}{l_1} - 1\right) \Big/ F,$$

where the Σ extends to the whole system,

or
$$d = \sum \frac{d_2}{l_1/l_1 \,.\, l_3/l_1} + \left(1 - \frac{l_1}{l}\right) \Big/ F \quad \ldots\ldots \text{XXII}b$$

To find the plane conjugate to the plane at the centre of curvature of any surface, say (5)

$$d = \sum_2^4 \frac{d_2}{l_1/l_1 \,.\, l_3/l_1} + \frac{r_5}{n_6 \,(l_5/l_1 \,.\, l_6/l_1)}$$

where $\dfrac{l_6}{l_1} = \dfrac{l_5}{l_1} - \dfrac{r_5}{n_6} \,.\, F_{1.3.5}$

that is
$$d = \frac{d_2}{l_1}\frac{l_3}{l_1} + \frac{d_4}{l_3}\frac{l_5}{l_1} + \frac{\dfrac{r_5}{n_6}}{\dfrac{l_5}{l_1}\left(\dfrac{l_5}{l_1} - \dfrac{r_5}{n_6} \,.\, F_{1.3.5}\right)}$$

$$= \frac{d_2}{l_1}\frac{l_3}{l_1} + \frac{d_4}{l_3}\frac{l_5}{l_1} + \frac{1}{\dfrac{l_5}{l_1}\left(n_6 R_5 \dfrac{l_5}{l_1} - F_{1.3.5}\right)} \quad \ldots \text{XXIII}b$$

where R_5 is written for $\dfrac{1}{r_5}$.

Corresponding relations will hold good if the ray for which l_1, l_3, etc. are calculated is not parallel to the axis. If this ray would meet the axis in medium (0) at a distance u beyond the vertex of the surface (1) and we write

$$U_1 = \frac{1}{u_1}$$

we have

$$l_3 = l_1 - l_1 d_2 (F_1 + U_1) \qquad F_{1,3} = F_1 + U_1 + l_3 F_3$$

and the other relations of I are unaltered, but the quantities $F_{1,3}$, etc., . . . F no longer represent equivalent-focal powers.

In this case

$$\frac{u_6'}{l_1 l_6'} = \sum_2^4 \frac{d_2}{l_1 l_3} + \frac{u_6}{l_5 l_6} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{XXIV}b$$

where l_6' is the intercept in medium (o) of the plane conjugate to (6) u_6' is the distance of this plane from surface (1) and $l_6' = l_1 - l_1 u_6' U_1$ is the relation connecting l_6' and u_6' with the quantities above . . XXV*b* we have

$$\frac{u_6}{1 - u_6' U_1} = \sum_2^4 \frac{d_2}{l_1/l_1 \,.\, l_3/l_1} + \frac{u_6}{l_5/l_1 \,.\, l_6/l_1}$$

$$= \sum_2^4 \frac{d_2}{l_1/l_1 \,.\, l_3/l_1} + \frac{u_6}{l_5/l_1 - (l_5/l_1 - u_6 F_{1.3.5})} \quad . \quad . \quad . \quad \text{XXVI}b$$

To find the Principal Planes let u_0 and u_7 be distances from the first and last surfaces of the Principal Planes of Incidence and Emergence and let l_0 and l_7 be the intercepts in these planes; then $l_0 = l_7$, and we have

$$\frac{u_0}{l_1 l_0} = \sum \frac{d_2}{l_1 l_3} + \frac{u_7}{l_1 l_7}$$

and

$$1 - u\ U_1 = \frac{l^0}{l_1} = \frac{l_7}{l_1} = \frac{l}{l_1} - u_7 F.$$

$$\frac{u_0}{1 - u_0 U_1} = \sum \frac{d_2}{l_1/l_1 \,.\, l_3/l_1} + \frac{u_7}{l/l_1 \,.\, l\ /l_1}$$

$$= \sum \frac{d_2}{l_3/l_1 \,.\, l_3/l_1} + \frac{l/l_1 - 1 + u_0 U_1}{l/l_1 \,.\, F\,(1 - u_0 U_1)}$$

giving

$$u_0 = u_1 \frac{\left(1 - \frac{l_1}{l}\right) f + \sum \frac{d_2}{l_1/l_1 \,.\, l_3/l_1}}{\left(u_1 - \frac{l_1}{l}\right) f + \sum \frac{d_2}{l_1/l_1 \,.\, l_3 l_1}}$$

or

$$\frac{u_0}{u_1 - u_0} = \frac{\left(1 - \frac{l_1}{l}\right) f + \sum \frac{d_2}{l_1/l_1 \,.\, l_3/l_1}}{u_1 - f}$$

or

$$\frac{F - U_1}{U_0 - U_1} = \left(1 - \frac{l}{l_1}\right) + \sum \frac{d_2}{l_1/l_1 \cdot l_3/l_1} F.$$

$$U_0 - U_1 = \frac{F - U_1}{1 - \frac{l}{l_1} + \sum \frac{d_2}{l_1/l_1 \cdot l_3/l_1} F.} \quad \ldots \quad \text{XXVIIb}$$

The apparent distance of any point may also be expressed in terms of q and a in an exactly similar way.

The equations are exactly the same as those above except that

$n_6\, C_5\, G_6$ is used instead of	$B_5\, G_6$
$n_6\, D_6\, G_6$	$D_6\, G_6$
$n_4\, C_5\, C_3 \equiv d_4$	$B_3\, B_5 = d_4$
q_5	l_5

The relations are

$$\frac{n_2\, C_1\, G_2}{q_1\, n_2\, D_2\, G_2} = \frac{a_2}{q_1\, q_3} + \frac{a_4}{q_3\, q_5} + \frac{n_6\, C_5\, G_6}{n_6\, D_6\, G_6\, q_5} \quad \ldots \quad \text{XXa}$$

the apparent distance is given by

$$\frac{a}{q_1\, q_1} = \sum \frac{a_2}{q_1\, q_3} + \frac{t_6}{q_5\, q_6} \quad \ldots \quad \text{XXIa}$$

where $q_6 = n_6\, D_6\, G_6$ and $t_6 = n_6\, C_5\, G_6$

and $q_6 = q_5 - q_1\, t_6\, P_{1.3.5}$

The Nodal Point may be obtained from

$$a = \sum \frac{a_2}{(q_1/q_1 \cdot q_3/q_1)} + \left(1 - \frac{q_1}{q}\right) \Big/ P. \quad \ldots \quad \text{XXIIa}$$

where q and P relate to the whole system.

The apparent distance relation may be used to find the connection between the intercepts on various surfaces when various object distances are used.

Let l_5 be the intercept on surface (5) of a parallel incident beam, l_5' be the intercept on this same surface of a parallel beam incident on the last surface, L_5 the intercept on this surface of a ray from an object at a distance

$$u_0 = \frac{1}{U_0}$$

Let the image of (5) in the first medium be at a distance d_0 from the first surface, and let the intercepts of the three rays on this plane be l_0, l' and L_0 respectively, and the intercepts on the first surface be l_1, l_1', L. Then

$$\frac{l_0}{l_5} = \frac{l_0'}{l_5'} = \frac{L_0}{L_5}$$

also

$$l_0 = l_1$$

$$\frac{l_1' - l_0'}{d_0} = \frac{l_1'}{v} \text{ where } v \text{ is the back focussing distance}$$

$$\frac{L_1 - L_0}{d_0} = \frac{L_1}{u_0} \quad \therefore \quad d_0 = v\,\frac{l_1' - l_0'}{l_1'}$$

$$\therefore \quad L_5 = L_0\frac{l_5}{l_0} = l_5\,\frac{L_1 - \dfrac{d_0}{u_0}L_1}{l_1}$$

$$= \frac{L_1\,l_5}{l^1}\left(1 - \frac{d_0}{u_0}\right)$$

$$\frac{L_5}{L_1} = \frac{l_5}{l_1}\left(1 - \frac{d_0}{u_0}\right)$$

$$\frac{l_5'}{l_1'} = \frac{l_5}{l_1}\left(1 - \frac{d_0}{v}\right)$$

$$\frac{\dfrac{L_5}{L_1} - \dfrac{l_5}{l_1}}{\dfrac{l_5}{l_1}} = -\frac{d_0}{u_0}$$

$$\frac{\dfrac{l_5'}{l_1'} - \dfrac{l_5}{l_1}}{\dfrac{l_5}{l_1}} = -\frac{d_0}{v} \qquad \text{* XXVIII}b$$

$$\left(\frac{L_5}{L_1} - \frac{l_5}{l_1}\right)u_0 = \left(\frac{l_5'}{l_1'} - \frac{l_5}{l_1}\right)v$$

$$\frac{L_5}{L_1} = \frac{l_5}{l_1}\left(\frac{u_0 - v}{u_0}\right) + \frac{l_5'}{l_1'}\cdot\frac{v}{u_0}$$

$$= \frac{l_5}{l_1}(1 - v\,U_0) + \frac{l_5'}{l_1}\frac{U_0}{F}$$

* The relations XXVIII*b* give expressions for the apparent distance which are often useful.

if the plane (1) be the Principal Plane

$$\frac{L_5}{L_1} = \frac{l_5}{l_1}\left(1 - \frac{U_0}{F}\right) + \frac{l_5'}{l_1}\frac{U_0}{F} \quad \ldots\ldots\ldots \text{XXIX}b$$

Again exactly analogous relations hold between the quantities q when calculated from a different ray.

The equivalent power of a system can also be expressed as—

$$\begin{aligned} F &= l_2{}^1F_{1\cdot3\cdot5} + l_5F_{5\cdot7\cdot9} \\ &= l_5{}^1F_{1\cdot3\cdot5} + l_5{}^5F_{5\cdot7\cdot9} \end{aligned}$$

This is very convenient for symmetrical systems.

In this case

$$F = 2lF'$$

where F' relates to the partial system and l is taken on the symmetrical plane.

This can be applied most simply to combined refracting and reflecting systems by treating the mirror as two elements of equal power.

And again exactly analogous relations exist in terms of q and P.

ABSTRACTS.

"ABSOLUTE SYSTEM OF COLOURS." W. Ostwald. (ZEIT. PHYSIK. CHEM. 91, pp. 129-142, 29th February, 1916.)

At present a particular colour can practically be ordered only to sample. The author considers that the difficulties, due to the impossibility of absolute measurements, might be overcome by basing measurements on 3 variables: purity of colour, brightness and tint. As regards purity, each colour may be regarded as a mixture of a colour and a grey constituent; the ratio of the two is the purity. Every coloured object, painted paper, *e.g.*, will absorb part of the incident light and reflect another part. Since the colour is never pure, but mixed with white light, the reflected part consists of two portions, the one due to the colour, the other due to the white light. When the object is illuminated by a spectrum a very bright portion and a very dark portion will stand out. The brightest portion will mark the spot where the colour agrees with the superposed spectrum. Taking the brightness of absolute white as unit, the brightness at that spot h_1 will be a proper fraction, and $(1-h_1)$ will represent absorbed light. At the darkest spot, corresponding to the complementary colour, the object will not reflect any light, but the white light will reflect, and this component will be h_2. These measurements probably will not require a spectrum, but may be performed with the aid of colour-filters. Now the light f, due to the pure colour, the white light, w, and the light absorbed, s, must be in the simple relation: $f+w+s=1$, and the author deduces 2 equations for purity and brightness which would allow of their determination. The third variable, the colour tint, depends directly upon physiological features and the individual, and further upon the light in which the test is performed; the determination of this variable is, moreover, affected by other factors. The author proposes to perform the test in daylight. Imagining a colour circle, on the circumference of which the colours are arranged in arcs, so that complementary colours are at the ends of diameters, he would divide this circle into 100 divisions, and start from the brightest yellow, represented by a 4 per cent. solution of calcium picrate on paper. No. 10 will be dark chrome yellow, No. 20 red lead, No. 32 carmine, No. 40 red-violet, No. 53 ultramarine, No. 62 Prussian blue, No. 70 chrome hydroxide; for the greens beyond No. 70 definite pigments are so far wanting. The first task in an analysis would be to determine, with the aid of a colour-mixing device, what colour of the colour circle, which combined with the colour under test, will give neutral grey. The author has obtanied concordant results in various tests, but does not give particulars.

"NEW RESEARCHES ON THE THEORY OF COLOUR." W. Ostwald. (PHYSIK. ZEIT. 19, pp. 322-332 and 352-364, 15th July and 1st August, 1916.)

Deals with similar problems in a less general way. H. B.

"LUMINESCENCE MEASUREMENTS." N. E. Dorsey. (JOURN. WASHINGTON ACAD. SCIENCES, 4th January, 1917.)

At the Optical Society's Exhibition on 11th January, J. Walsh described his method of determining the luminescence of radium-painted dials as used at the National Physical Laboratory. The U.S. Bureau of Standards makes use of a photometer box, about 25 cm. square, 2 or 3 m. long, within which a carriage supporting a lamp (1.5 or 2 candles) runs centrally when pulled by a cord. The front of the box is closed by an opal glass screen 2 or 3 cm. long, which is backed by a colour filter so as to match the tints of the transmitted light and of the luminous object, which is either a painted card or material placed in a small glass tube (painted black over two-thirds of its outer circumference). The card or tube is placed against the screen, the unpainted part of the tube facing the observer, and the tube is then turned so as to reduce the visible dark edge of the tube as much as possible. The carriage is then moved until the tube disappears on the screen. The material must not be exposed to any light; and to make the eye sensitive, the observer should keep in the dark for at least 15 minutes before the test. Even then the Purkinge effect causes difficulties at low intensities. Two equally bright adjoining patches will, owing to the Purkinge effect, not appear at equal brightness, especially when at low intensity, if viewed from a different distance. When several bands scattered over the spectrum are to be examined, the effect may make the colour matching very difficult. When there is only one band the photometer can suitably be combined with other appliances. The surface brightness is expressed in micro-lamberts, one micro-lambert being the brightness of a perfectly diffusing and completely reflecting white surface when illuminated by 1 candle at 10 m. distance. If, therefore, the lamp be at 70.5 cm. from the screen when lamp and specimen appear equally bright, and the surface brightness of the screen be 3.96 micro-lamberts, the brightness of the specimen will be $3.96 \times 1.38/0.705^2 = 11$ micro-lamberts, the factor 1.38 representing the candle power of the tungsten lamp.

H.B.

'AUTOMATIC MECHANISM FOR WINDING-UP TELESCOPE CLOCKWORKS." A. Hnatek. (ZEIT. FÜR INSTRUMK. 36, pp. 295-297, December, 1916.)

With the aid of electric motors, clockworks may automatically be kept wound-up so that the driving weight remains at the same level. For this purpose the driving gear of the Rotschild-Coudè telescope of the Vienna University Observatory was provided with a pair of conical pulleys, interposed between the motor and the winding mechanism; the belt

resting on these pulleys was shifted by the descending or ascending weight. This arrangement did not give satisfaction because, for want of space, the motor had practically to be mounted in the open, and the pulleys could not be made sufficiently long to meet the resulting temperature and speed fluctuations. The author, therefore, adopted the following arrangement, which works well :—A rope is attached to the clock weight and kept taut by a small counter-weight ; when the weight ascends, the rope moves to the right. On the rope rides a fixed clamp which turns a lever over to the right or left as the rope travels. The pivot of this lever is also the pivot of a sector which is turned to an arc on its upper edge, but cut off at an obtuse angle below. Near the top of the sector is a slot (also an arc) in which a pin glides which is fixed on the lever. One contact spring bears against the upper edge of the sector, the other against one of the two edges of the sector below. The two springs are part of the electric circuit of the motor, the upper spring being carried by an arm which projects from the lever ; insulations, it should be mentioned, are inserted between this spring and the arm, as well as between the upper end of the lever and the clamp. When the rope moves (with the weight) over to the right, the lever and the sector are together tilted until the lower spring loses contact. The circuit is then broken, and the weight descends until the lower spring comes in contact with the other edge and closes the circuit again. With this simple arrangement the operations of the motor are thus intermittent ; the vertical play of the clock weight amounts to 10 cm.

H.B.

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